MODELING THE APPEARANCE OF THE ROUND BRILLIANT CUT DIAMOND: AN ANALYSIS OF BRILLIANCE

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Of the "four C's," cut has historically been the most complex to understand and assess. This article presents a three-dimensional mathematical model to study the interaction of light with a fully faceted, colorless, symmetrical round-brilliant-cut diamond. With this model, one can analyze how various appearance factors (brilliance, fire, and scintillation) depend on proportions. The model generates images and a numerical measurement of the optical efficiency of the round brilliant—called weighted light return *(WLR)—which approximates overall brilliance. This article examines how WLR values change with variations in cut proportions, in particular crown angle, pavilion angle, and table size. The results of this study suggest that there are many combinations of proportions with equal or higher WLR than "Ideal" cuts. In addition, they do not support analyzing cut by examining each proportion parameter independently. However, because brilliance is just one aspect of the appearance of a faceted diamond, ongoing research will investigate the added effects of fire and scintillation.*

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The quality and value of faceted gem diamonds are often described in terms of the "four C's": carat weight, color, clarity, and cut. Weight is the most objective, because it is measured directly on a balance. Color and clarity are factors for which grading standards have been established by GIA, among others. Cut, however, is much less tractable. Clamor for the standardization of cut, and calls for a simple cut grading system, have been heard sporadically over the last 25 years, gaining strength recently (Shor, 1993, 1997; Nestlebaum, 1996, 1997). Unlike color and clarity, for which diamond trading, consistent teaching, and laboratory practice have created a general consensus, there are a number of different systems for grading cut in round brilliants. As discussed in greater detail later in this article, these systems are based on relatively simple assumptions about the relationship between the proportions and appearance of the round brilliant diamond. Inherent in these systems is the premise that there is *one* set (or a narrow range) of preferred proportions for round brilliants, and that any deviation from this set of proportions diminishes the attractiveness of a diamond. In this article, we present and discuss our findings with regard to the rather complex relationship between cut proportions and brilliance.

Diamond manufacturing has undergone considerable change during this century. For the most part, diamonds are cut within very close proportion tolerances, both to save weight while maximizing appearance and to account for local market preferences (Caspi, 1997). As shown in figure 1 and table 1, however, differences in proportions can produce noticeable differences in appearance in round-brilliant-cut diamonds. Within this single cutting style, there is substantial debate—and some strongly held views—about which proportions yield the best face-up appearance (Federman, 1997). Yet face-up appearance depends as well on many intrinsic physical and optical properties of diamond as a

Figure 1. These round-brilliant-cut diamonds illustrate how cut affects face-up appearance. Of the three larger stones (1.07–1.50 ct), the one on the lower right (F color) is obviously less bright than the other two (above, H color, and lower left, E color). All three 0.35–0.38 ct diamonds in the inset are brighter, on average; but the F-color diamond on the lower right, which could be marketed as an "Ideal" cut, is less bright than the F- and G-color stones with larger tables and small crown angles. See table 1 for the proportions and other data on these diamonds. Photo © GIA and Tino Hammid.

material, and on the way these properties govern the paths of light through the faceted gemstone. (Also important are properties particular to each stone, such as polish quality, symmetry, and the presence of inclusions.)

Diamond appearance is described chiefly in terms of *brilliance* (white light returned through the crown), *fire* (the visible extent of light dispersion into spectral colors), and *scintillation* (flashes of light reflected from the crown). Yet each of these terms represents a complex appearance concept that has not been defined rigorously, and that cannot be expressed mathematically without making some assumptions and qualifications (see below).

Despite the widespread perception in the trade that diamond appearance has been extensively addressed, there is limited information in the literature, and some aspects have never been examined. Several analyses of the round brilliant cut have been published, starting with Wade (1916). Best known are Tolkowsky's (1919) calculations of the proportions that he believed would optimize the appearance of the round-brilliant-cut diamond. However, Tolkowsky's calculations, as well as most others since then, involved two-dimensional images as graphical and mathematical models. These were used to solve sets of relatively simple equations that described what was considered to be the brilliance of a polished round brilliant diamond. (Tolkowsky did include a simple analysis of fire, but it was not central to his model and it will not be discussed at any length in this article.) For the most part, the existing cut grading systems are based on Tolkowsky's research.

We believe that diamond cut, as a matter of such importance to the trade, deserves a more thorough and thoughtful investigation. The issues raised can only be resolved by considering the complex combination of physical factors that influence the appearance of a faceted diamond (i.e., the interaction of light with diamond as a material, the shape of a given polished diamond, the quality of its surface polish, the type of light source, and the illumination and viewing conditions), and incorporating these into an analysis of that appearance.

The initial goal of this research project was to develop a theoretical model for the interaction of light with a faceted diamond that could serve as the basis for exploring many aspects of the effect of cut on appearance. Computer graphics simulation techniques were used to develop the model presented here, in conjunction with several years of research

TABLE 1. Proportions and calculated WLR values for the diamonds photographed in figure 1.

Position	Color	Weight (ct)	Table size (%)	Crown angle (°)	Pavilion angle (°)	Calculated WLR[a]
Main photo						
Top	H	1.21	62	29.4	41.7	0.279
Lower right	F	1.50	63	39.8	41.7	0.257
Lower left	E	1.07	57	34.6	40.9	0.282
Inset						
Top	G	0.38	60	26.5	42.6	0.288
Lower right	F	0.35	56	34.7	41.2	0.281
Lower left	F	0.35	59	27.0	41.4	0.290

[a] *WLR, our metric for overall brightness, is calculated from the given crown angle, pavilion angle, and table size, using our standard reference proportions (given in table 4) for the other five parameters.*

on how to express mathematically the interaction of light with diamond and also the various appearance concepts (i.e., brilliance, fire, and scintillation). Our model serves as a general framework for examining cut issues; it includes mathematical representations of both the shape of a faceted diamond and the physical properties governing the movement of light within the diamond. We plan to analyze the appearance aspects one at a time and then, ultimately, assemble the results in order to examine how proportions affect the balance of brilliance, fire, and scintillation.

The *general* mathematical model presented in this article uses computer graphics to examine the interaction of light with a standard (58 facet) round-brilliant-cut diamond with a fully faceted girdle. For any chosen set of proportions, our model can produce images and numerical results for an appearance concept (by way of a mathematical expression). To compare the appearance concepts of brilliance, fire, and scintillation in round brilliants of different proportions, we need a quantity to measure and a relative scale for each concept. A *specific* mathematical expression (with its built-in assumptions and qualifications) that aids the measurement and comparison of a concept such as brilliance is known as a *metric.* In this study, we derived a metric for brilliance that quantifies the amount of light returned from a modeled diamond for averaged illumination and viewing arrangements, as described below. Although other factors (e.g., bodycolor or inclusions) may also influence how bright a particular round brilliant appears, light return is an essential feature of diamond brilliance.

In future reports on this project, we plan to address how fire and scintillation are affected by proportions. We also intend to examine how symmetry, lighting conditions, and other factors affect all three of these appearance concepts. The overall goal of this research is to provide a comprehensive understanding of how cut affects the appearance of a faceted diamond.

BACKGROUND

Early History. Diamond faceting began in about the 1400s and progressed in stages toward the round brilliant we know today (see Tillander, 1966, 1995). In his early mathematical model of the behavior of light in fashioned diamonds, Tolkowsky (1919) used principles from geometric optics to explore how light rays behave in a prism that has a high refractive index. He then applied these results to a two-dimensional model of a round brilliant with a knife-edge girdle, using a single refractive index (that is, only one color of light), and plotted the paths of some illustrative light rays.

Tolkowsky assumed that a light ray is either totally internally reflected or totally refracted out of the diamond, and he calculated the pavilion angle needed to internally reflect a ray of light entering the stone vertically through the table. He followed that ray to the other side of the pavilion and found that a shallower angle is needed there to achieve a second internal reflection. Since it is impossible to create substantially different angles on either side of the pavilion in a symmetrical round brilliant diamond, he next considered a ray that entered the table at a shallow angle. Ultimately, he chose a pavilion angle that permitted this ray to exit through a bezel facet at a high angle, claiming that such an exit direction would allow the dispersion of that ray to be seen clearly. Tolkowsky also used this limiting case of the ray that enters the table at a low angle and exits through the bezel to choose a table size that he claimed would allow the most fire. He concluded by proposing angles and proportions for a round brilliant that he believed best balanced the brilliance and fire of a polished diamond, and then he compared them to some cutting proportions that were typical at that time. However, since Tolkowsky only considered one refractive index, he could not verify the extent to which any of his rays would be dispersed. Nor did he calculate the light loss through the pavilion for rays that enter the diamond at high angles.

Over the next 80 years, other researchers familiar with this work produced their own analyses, with varying results (see table 2). It is interesting (and somewhat surprising) to realize that despite the numerous possible combinations of proportions for a standard round brilliant, in many cases each researcher arrived at a *single* set of proportions that he concluded produced an appearance that was superior to all others. Currently, many gem grading laboratories and trade organizations that issue cut grades use narrow ranges of proportions to classify cuts, including what they consider to be best (table 3).

Several cut researchers, but not Tolkowsky, used "Ideal" to describe their sets of proportions, which vary significantly, as seen in table 2. Today, in addition to systems that incorporate "Ideal" in their names, many people use this term to refer to measurements similar to Tolkowsky's proportions, but with a somewhat larger table (which, at the same crown angle, yields a smaller crown height percentage). This is what we mean when we use "Ideal" in this article.

Recent Appearance Models and Measurements. We found thorough descriptions of three computer models of round brilliant diamonds in the literature. One model explored light return numerically (Tognoni, 1990); another produced a number of monochromatic images, each using a different refractive index, and some numerical output (Astric et al., 1992); but the third (Dodson, 1979) was similar to our work in several ways. Using a three-dimensional model of a fully faceted round brilliant diamond, Dodson devised metrics for brilliance, fire, and "sparkliness" (scintillation). His mathematical model employed a full sphere of approximately diffuse illumination centered on the diamond's table. His results were presented as graphs of brilliance, fire, and sparkliness for 120 proportion combinations. They show the complex interdependence of all three appearance aspects on pavilion angle, crown height, and table size. His model (as well as that of Shannon and Wilson, as best we can determine from the little published on it [Lawrence, 1998; Shor, 1998]) is distinct from ours in that all rays emerging from the diamond were weighted equally. Three of Dodson's results are given in table 2.

There are also computer-aided-design (CAD) software programs for creating gemstone cuts and analyzing the effect of cut on the appearance of the finished gemstone. (One of these, Gem-Cad, is marketed by Gemsoft Enterprises, Austin, Texas.)

These computer programs and mathematical models use ray-tracing algorithms to produce visual images of gemstones or numerical data about their appearance, or both. However, each of the programs described above excludes one or more of the starting assumptions that we employ here (e.g., wavelength-dependent refractive index, accounting for secondary rays, weighing observer angles; see below and Box A). Because of these differences, our computer graphics program is not directly comparable to these other programs. However, the optimal proportions predicted by those models can be assessed and compared using our metric for brilliance.

Commercial services are currently available that claim to measure the brilliance of fashioned stones. The measurements of brilliance provided by Diamond Profile (Portland, Oregon) are based on digital video images through the crown of the diamond under a few controlled lighting conditions, which are then combined to generate graphic results for that particular stone (Gilbertson and Walters, 1997; Gilbertson, 1998).

DESCRIPTION OF OUR MODEL

In general, within a mathematical model, all of the factors we consider important to diamond appearance—the diamond itself, its proportions and facet arrangement, and the lighting and observation conditions—can be carefully controlled, and fixed for a given set of analyses. Such control is nearly impossible to achieve with actual diamonds. Furthermore, with this model we can examine thousands of sets of diamond proportions that would not be economically feasible to create from diamond rough. Thus, use of a model allows us to explore how cut proportions affect diamond appearance in a more comprehensive way than would be possible through observation of actual round brilliants. However, every mathematical model incorporates some assumptions, and these built-in conditions affect the nature of the results. (The modeled diamond used in Tolkowsky's [1919] analysis, for example, was two-dimensional and had a knife-edge girdle, which limited the number and paths of light rays he could consider.)

Real diamonds will inevitably differ from the model conditions because of inclusions, symmetry deviations, and the like. Nevertheless, a theoretical model provides a goal to reach toward: What is the best result—best brilliance, best fire, best balance of the two, best scintillation, best weight retention, best combination—that can be achieved from a particular piece of rough? In addition, a theoretical understanding of the behavior of light in a faceted diamond could help in the design of any instrument intended to measure optical performance in real diamonds.

Finally, a model of the interaction of light with a faceted diamond can be used to compare and contrast different metrics and different lighting and observation conditions, as well as evaluate the dependence of those metrics on proportions, symmetry, or any other property of diamond included in the model. In the following sections, we present the assumptions and methods on which our model is based, and introduce our metric for brilliance.

Assumptions and Methods. The mathematical model presented here creates a fresh structure for examining nearly all aspects of the influence that cut has on a diamond's appearance. Box A provides the assumptions on which the model is based: a detailed list of the physical properties included in the model, a mathematical description of the proportions of the round brilliant, and a description of the lighting condition used in this study. The inclusion of these many physical properties distinguishes this model from previous work, and the details of the lighting conditions affect the specific numerical values we present here. The model traces rays from the modeled light source through a mathematical representation of a round brilliant of any chosen proportions (referred to hereafter as the "virtual" diamond) to produce two kinds of results: (1) digital images of the virtual diamond, and (2) a numerical evaluation of an appearance concept (in this case, brilliance).

A digital image (see, e.g., figure 2), drawn from the perspective of our choice, is a two-dimensional array of picture elements (pixels), each of which comprises a small area of the virtual diamond. We traced up to one million rays of various colors for each pixel in an image, to obtain convergence of the color and total brightness for that small area. As the computer traces the first few hundred rays, randomly selecting wavelengths and angles of incidence, the computed brightness and color for a given pixel change rapidly. Eventually, when enough different directions and wavelengths have been traced, the computed brightness and color settle down, or converge, to particular values, and tracing more rays does not change these values. The resolution of the image depends on our choice of the number of pixels to compute for a particular image size. For most of the images presented in this article, we calculated the color and brightness of 65,536 pixels, requiring up to 65,000,000,000 traced rays.

The computer program employed is not a commercial product, but was written specifically for this work by the first author. It was written in C, a scientific programming language. The program has been run on a Pentium personal computer, on two models of Digital Equipment Corp.'s Alpha workstation, and on a dual Pentium II. If the convergence thresholds and choice of resolution are maintained, the hardware used to run the program does not alter the results. The accuracy of the program, in general and on different kinds of hardware, was verified with a simple test problem for which we had computed a result manually. Further details of the ray tracing and computational methods will be given by Hemphill et al. (in preparation). These techniques extend the methods described by Foley (1996).

Defining Metrics: Brilliance. Our aim is to use this model to explore how brilliance, fire, and scintillation vary with the proportions of a round brilliant diamond. We begin with brilliance for several reasons. First, brilliance is the aspect of diamond appearance that is most immediately noticed. Second, it is an aspect for which the desired outcome is obvious: Bright is good and dark is not. Last, most of the previous work investigating cut focused on brilliance (see references in table 2), and it is this work that has fueled the current trade debate about cut.

One advantage of using a computer model is the capability it gives us to examine thousands of proportion variations. To make sense of so much data, however, we needed to define a metric for brilliance, and use it to compare the performance of the different proportion combinations. The *GIA Diamond Dictionary* (1993, p. 28) defines brilliance as the "intensity of the internal and external reflections of white light from the crown. . . ." A variety of mathematical expressions can be created to describe such light return. Each expression requires explicit or implicit assumptions about what constitutes brilliance and about light sources, viewing geometry, response of the human eye, and response of the human brain. As an example of this last consideration, should a mathematical definition of brilliance represent one viewing geometry—that is, a

TABLE 2. Superior proportions for a round-brillant-cut diamond, as suggested by previous investigators.

Name	Investigator	Year	Table size (%)	Crown angle (Crown height)	Pavilion angle (Pavilion height)	Total depth (%)	Girdle thickness (%)	Calculated WLR[a]
American Ideal	Wade	1916	45.3	35°(19.2%)	41°(43.5%)	62.7	0	0.266
(None)	Tolkowsky[b]	1919	53	34.5° (16.2%)	40.75° (43.1%)	59.3	0	0.281
Ideal	Johnsen	1926	56.1	41.1° (19.2%)	38.7° (40%)	59.2	nd[c]	0.252
Ideal	Rösch	1926, 1927	56	41.1° (19%)	38.5° (40%)	59	0	0.251
Normal Universal	Stoephasius	1931	54	38° (18%)	36.5° (37%)	56[d]	1	0.262
(None)	Stoephasius	1931	52	41° (21%)	39.4° (41%)	64	2	0.248
(None)	Stoephasius	1931	50	43.8°(24%)	41.4° (44%)	72	4	0.216
Total Reflection	Maier	1936, 1938	nd	40.8°–41.3°	38.6°	nd	nd	0.237–0.251[a]
Ideal	Bergheimer	1938	nd	41.1°	38.7°	nd	nd	0.238–0.243[a]
Ideal	Eppler	1933, 1938	56	41.1° (19%)	38.5° (40%)	59	2	0.251
Practical Fine	Eppler	1939	56	33.2° (14.4%)	40.8° (43.2%)	57.6	nd	0.284
Ideal Type I	Eppler	1939, 1940	56.1	41.1° (19.2%)	38.7° (40%)	59.2	2	0.252
Ideal Type II	Eppler[b]	1940	57.1	33.1° (14%)	40.1° (42.1%)	57.6	1.5	0.281
Ideal	Eppler and Klüppelberg	1940	56.1	41.1° (19.2%)	38.7° (40%)	59.2	nd	0.252
Practical Fine I	Eppler and Klüppelberg	1940	55.3	35.6° (16%)	38.6° (39.9%)	55.9	2	0.274
Practical Fine II	Eppler and Klüppelberg	1940	57.1	33.1° (14%)	40.1° (42.1%)	56.1	1.5	0.281
Practical Fine III	Eppler and Klüppelberg[b]	1940	69	32.8° (10%)	41.7° (44.6%)	54.6	1	0.264
(None)	Parker (cited by Eppler, 1973)	1951	55.9	25.5° (10.5%)	40.9° (43.4%)	53.9	nd	0.297
Practical Fine	Schlossmacher	1969	56.4	33.2° (14.4%)	40.8° (43.2%)	57.6	nd	0.284
Standard Cut	ScanDN[e]	1979	57.5	34.5° (14.6%)	40.75° (43.1%)	57.7	2 to 3	0.282
Brilliance Design	Suzuki[f]	1970	56	41.1° (19%)	38.7° (40%)	59	nd	0.252
Dispersion Design	Suzuki[f]	1970	58	48.6° (23%)	38.9° (40%)	63[d]	nd	0.205
(None)	Elbe	1972	nd	(14.6%)	47° (53.7%)	68.3	nd	nc[g]
Optical Symmetrical	Eulitz	1972	56.5	33.6° (14.45%)	40.8° (43.15%)	59.1	1.5	0.283
(Brightest)	Dodson[b,f]	1979	40	26.5° (15%)	43°	nd	nd	0.277
(Most fire)	Dodson[b,f]	1979	60	26.5° (10%)	43°	nd	nd	0.287
(Most sparkliness)	Dodson[b,f]	1979	50	31.0° (15%)	52°	nd	nd	0.247
Australian Ideal	Connellan and Pozzibon	1984	56	33.75° (14.3%)	39.66° (41.45%)	55.75	nd	0.281
Modern Ideal	Watermeyer	1991	61	34.0°	41.0°	nd	nd	0.279
(None)	Shannon and Wilson[f] (Shor, 1998)	1998	61	32°	(41%)	nd	nd	0.275
(None)	Shannon and Wilson[f] (Shor, 1998)	1998	57	32°	(42%)	nd	nd	0.281
(None)	Shannon and Wilson[f] (Shor, 1998)	1998	58	33.5°	(43.1%)	nd	nd	0.282
(None)	Shannon and Wilson[f] (Shor, 1998)	1998	50	33°	(46%)	nd	nd	0.279

[a] *WLR, our metric for overall brightness, is calculated from the given crown angle, pavilion angle, and table size, using our standard reference proportions (given in table 4) for the other five parameters. For Maier's "Total Reflection" and Bergheimer's "Ideal" cuts, where no table size was specified, we calculated WLR for tables ranging from 50% to 70%.*

[b] *Used broad illumination across the entire crown rather than the vertical illumination used by others.*

[c] *nd = not defined.*

[d] *The measurements given are not consistent with this depth percentage.*

[e] *Scandinavian Diamond Nomenclature Committee.*

[f] *Used three-dimensional analysis rather than the two-dimensional analyses used by other investigators.*

[g] *nc = not calculated (not enough information to calculate the WLR value).*

TABLE 3. Proportions for some of the cut grading systems for round brilliant diamonds used today.[a]

Cut grading system[b]	Category[a]	Table size (%)		Crown angle (°)		Pavilion angle (°)		Pavilion depth (%)	
		Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.
AGA	*1A	53	58	34.3	34.7	40.5[d]	40.8[d]	42.8	43.2
	1B	52.0–52.9	58.1–60.0	34.01–34.2	34.81–35.0	40.36–40.5[d]	40.9–41.0[d]	42.5–42.7	43.3–43.5
	2A	51.5–51.9	60.1–63.0	32.1–33.9	35.1–35.8	40.03–40.03[d]	41.08–41.3[d]	42.0–42.4	43.6–44.0
	3A	50.5–50.9	64.1–67.0	29.6–30.0	38.0–39.4	39.35–39.6[d]	42.0–42.3[d]	41.0–41.4	45.0–45.5
AGJ	* Excellent	53	58	33	35	40.0[d]	41.3[d]	42	44
	Very Good	52	63	32	36	40.0[d]	42.0[d]	42	45
	Good	50	67	30	38	39.3[d]	43.2[d]	41	47
AGS	*0	52.4	57.5	33.7	35.8	40.16[d]	41.2[d]	42.2	43.8
	1	51.4–52.3	57.6–59.5	32.7–33.6	35.9–36.3	40.16[d]	41.3–41.5[d]	42.2	43.9–44.3
	3	50.4–51.3	61.6–63.5	31.7–32.1	36.9–37.3	40.16[d]	41.3–41.5[d]	42.2	43.9–44.3
	5	49.4–50.3	65.6–67.5	30.7–31.1	37.9–38.3	39.1–39.4[d]	42.2–42.5[d]	40.7–41.1	45.4–45.8
AGT	* Excellent	53	60	33	35	40.7[d]	40.7[d]	43	43
	Very Good	52	64	32	36	40.0[d]	41.3[d]	42	44
	Good	51	68	30	38	39.3[d]	42.6[d]	41	46
CGL	* Excellent	52	58	33	35	40.0[d]	41.3[d]	42	44
EGL	* Exceptional	54	57	34	35	40.5	41.0	42.5	43.5
HRD	* Very Good	53	66	30.7	37.7	39.6[f]	42.2[f]	41.5[f]	45[f]
IGI	* Ideal	53	60	33	36	40.0[d]	41.3[d]	42	44
Rap	* Specification A	55	64	30	35	nd	nd	nd	nd
ScanDN	* Good	52	65	30	39	40	42	42	45

[a] *Asterisk (*) indicates top grade. Generally all parameters must be in the specified ranges for the stone to receive the top grade; variation in any parameter reduces the grade accordingly. Abbreviations: Max.= maximum, Min.= minimum, med.= medium, sl.= slightly, ex.= extra, v.= very.*

[b] *The full names and countries of the organizations are indicated below. For some organizations, only the top grade is provided. All of the information provided in this table was obtained from the respective organizations:*

AGA = As reported in Fine Make*, Accredited Gem Appraisers, 1997, U.S. (Class 1A and 1B = "American Ideal Cut," Class 2A = "International Fine Cut," Class 3A = "U.S. Domestic Average Cut." Values given for stones ≥0.5 ct)*

AGJ = Association of Gemological Laboratories, 1993, Japan

AGS = American Gem Society, 1997, U.S.

AGT = Association of Japan Gem Trust, 1995, Japan

CGL = Central Gem Laboratory, 1993, Japan

EGL = European Gemological Laboratory, U.S. (G. Sherman, pers. comm., 1997)

HRD = Hoge Raad voor Diamant, 1993, Belgium

IGI = International Gemological Institute, 1997, U.S.

"snapshot"—or an average over many viewing situations? We chose the latter approach.

Weighted Light Return. The metric we discuss in this article is called *weighted light return* (WLR); it is specific to each set of modeled diamond proportions with the chosen illumination. After examining a variety of possible metrics for brilliance, we developed WLR to best represent the way the experienced viewer sees a diamond, especially one mounted in jewelry, with lighting that illuminates the stone from all around without excessive glare or shadow.

The WLR is a weighted sum of the amount of light returned through the crown of the virtual diamond to all positions of observation above the girdle. Rather than using the total fraction of light returned through the crown for a fixed arrangement of the light source, diamond, and viewer, we weighed the relative importance of returned light

Total depth (%)		Girdle thickness (%)[a]		Calculated WLR[c]	
Min.	Max.	Min.	Max.	Min.	Max
58.7	62.3	thin (1.7%)	med. (3.0%)	0.281	0.283
58.3–58.69	62.31–62.99	thin (1.5%)	sl. thick (4.5%)	0.278	0.284
57.9–58.29	63.0–63.5	thin (1.0%)	sl. thick (4.5%)	0.271	0.286
57.0–57.49	63.81–64.1	v. thin (0.4%)	v. thick (7.0%)	0.245	0.289
59.2	62.4	thin	sl. thick	0.279	0.285
58	63.8	ex. thin	sl. thick	0.270	0.286
56.8	65.9	ex. thin	v. thick	0.248	0.289
nd[e]	nd	thin	sl. thick	0.275	0.284
nd	nd	v. thin	nd	0.271	0.285
nd	nd	nd	thick	0.262	0.286
nd	nd	nd	v. thick	0.250	0.288
59	63	thin	sl. thick	0.279	0.284
58	64	thin	thick	0.269	0.286
56	66	v. thin	v. thick	0.252	0.289
59.2	62.4	thin	sl. thick	0.278	0.285
nd	nd	thin	med.	0.284	0.284
55.5	63.9	thin	med.	0.262	0.288
58.5	62.5	thin	sl. thick	0.274	0.285
57.5	62.5	v. thin	thick	nc[g]	nc
nd	nd	v. thin	sl. thick	0.251	0.288

Rap = Rapaport Diamond Report, *©1998, U.S.*
ScanDN = *Scandinavian Diamond Nomenclature Committee,1979, Denmark*

[c] *Minimum and maximum WLR values were calculated for the entire range of properties listed. See text.*

[d] *Converted from pavilion depth using the formula given in Box A.*

[e] *nd = not defined.*

[f] *The given pavilion angle and pavilion depth do not correlate exactly.*

[g] *nc = not calculated (not enough information to calculate the WLR values).*

rays based on their exit direction. An experienced diamond observer assesses the diamond primarily on the basis of its face-up appearance, but also "rocks" the stone both to minimize the effect of glare and to consider the stone from various angles, with the views closest to vertical (face-up) weighing the most in this evaluation. We wanted the metric we chose to behave like this assessment. Therefore, we wanted the contribution from rays that emerged straight up to be much greater than that from rays that exited horizontally, with a smooth variation as the exit angle changed. We chose the square of the cosine function, applied to the exit angle measured from the vertical, as a weighting factor (figure 3). In contrast to this, both Dodson (1979) and Shannon and Wilson (Shor, 1998) considered all views of the diamond's crown to be equally important, and so they weighed much larger angles from the vertical far more heavily than our metric does (or than experienced observers do).

With this weighting function, we constrained the scale of the numbers for our metric between values of 0 and 1. For instance, if we could construct a virtual diamond in which all light that entered left the crown straight up, it would have a WLR equal to 1.000; but if all the light that entered left the crown at an angle of 25° to the horizontal, the WLR would be 0.179. If light only exited from the crown horizontally (or no light left through the crown), the WLR would be 0. Similarly, if half the light passed out of the crown at 45°, one fourth exited at 25°, and the remaining fourth was horizontal, the WLR would equal 0.294. This weighting function emphasizes the angle at which a light ray exits the virtual diamond, rather than which facet the ray exits.

Note that we excluded glare—that is, any light directly reflected from the top surface—from the WLR value (a difference from the *GIA Diamond Dictionary* definition of brilliance). We made this simple change in our computer program to guarantee that any trends in the WLR data were not simply due to an increased area of the crown acting like a front-surface reflector. However, this is also a reasonable change to the metric, since when experienced observers "rock" a diamond, they mentally correct for the effects of glare. (We also checked our results with glare included and found that although the WLR values increased across the whole range of proportions, the relative variation was essentially unchanged.)

We systematically explored the dependence of WLR on the eight proportion parameters that define the perfectly symmetrical round brilliant diamond (again, see Box A) by changing one or more of these parameters across the ranges given in table 4 and computing the WLR for each set of proportions. Although ideally we would have liked to examine all the interactions between WLR and the eight parameters, this was not practical given existing computer hardware. If we were interested in the co-variation of, say, 20 values for each of the eight parameters, we would require 20^8 =

BOX A:
BASIC DESCRIPTION OF OUR MODEL

We describe a faceted diamond as a convex polyhedron, a three-dimensional object with a surface that is bounded by flat planes and straight edges, with no indentations or clefts. The model requires that all surfaces be faceted, including the girdle, and currently excludes consideration of indented naturals or cavities. To date, we have focused our calculations on the round brilliant cut because of its dominant position in the market, but this model can be used for nearly any fully faceted shape. Our modeled round brilliant has mathematically perfect symmetry; all facets are perfectly shaped, pointed, and aligned. Also, all facet junctions are modeled with the same sharpness and depth.

Because our modeled round brilliant has perfect eight-fold symmetry, only eight numbers (parameters) are required to specify the convex polyhedron that describes its shape (figure A-1). (Modeling other shapes or including asymmetries requires additional parameters.) We defined these eight parameters as:

Crown angle	Angle (in degrees) between the bezel facets and the girdle plane
Pavilion angle	Angle (in degrees) between the pavilion mains and the girdle plane
Table size	Table width (as percent of girdle diameter)
Culet size	Culet width (as percent of girdle diameter)
Star length	The ratio of the length of the star facets to the distance between the table edge and girdle edge
Lower-girdle length	The ratio of the length of the lower-girdle facets to the distance between the center of the culet and girdle edge
Girdle thickness	Measured between bezel and pavilion main facets (the thick part of the girdle) and expressed as a percentage of girdle diameter. This differs from the typical use of the term *girdle thickness* (see, e.g., *GIA Diamond Dictionary*, 1993)
Girdle facets	Total number of girdle facets

Other proportions, such as the crown height, pavilion depth, and total depth (expressed as percentages of the girdle diameter) can be directly calculated from these eight parameters, using these formulas:

$$\text{Crown height} = \tfrac{1}{2}(100 - \text{table size}) \times \tan(\text{crown angle})$$

$$\text{Pavilion depth} = \tfrac{1}{2}(100 - \text{culet size}) \times \tan(\text{pavilion angle})$$

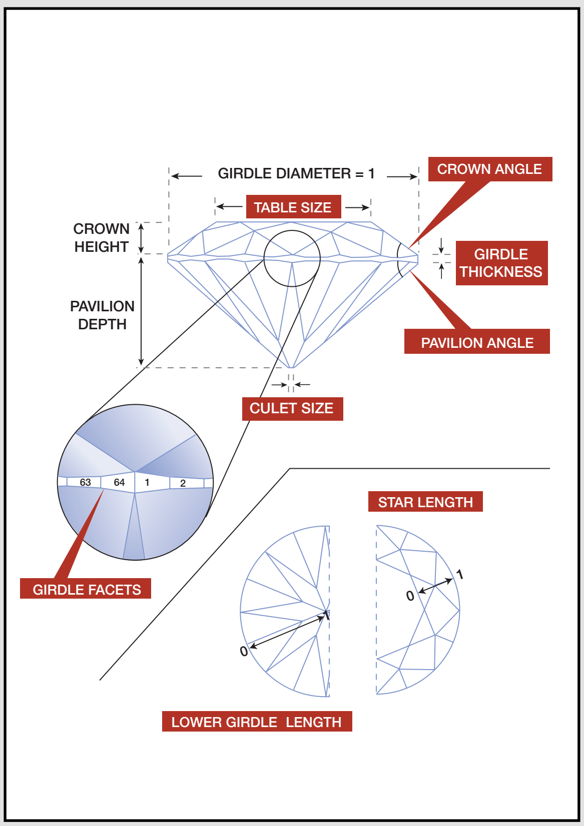
$$\text{Total depth} = (\text{Crown height} + \text{pavilion depth} + \text{girdle thickness})$$

For the results in this article, the diamond simulated in our calculations (called a "virtual" diamond) has no inclusions, is perfectly polished, and is completely colorless. It has a polished girdle, not a bruted one, so that the girdle facets refract light rays in the same way that other facets do. The virtual diamond has relative proportions but no absolute size—that is, no specific carat weight. (The principles governing the way light moves through a colorless diamond do not vary with size.)

We then chose a light source to illuminate our virtual diamond. Most of our results to date, and all the results presented here, used a diffuse hemisphere of even, white light (D65 daylight illumination) shining on the crown. Light rays from every position on the hemisphere point in every direction, not just toward the center of the stone (as in focused light). We selected this illumination condition to best average over the many different ambient light conditions in which diamonds are seen and worn, from the basic trading view of a diamond face-up in a tray next to large north-facing windows, to the common consumer experience of seeing a diamond worn outdoors or in a well-lit room. Diffuse illumination emphasizes the return of white light, although it is a poor lighting condition for examining other aspects, such as fire. This geometry also eliminated the need to consider the shadow that a viewer's head might cast on a diamond. (In addition, although many mountings, such as a Tiffany setting, allow some light to enter the diamond's pavilion, the amount of light coming from this direction has not been included in the model.)

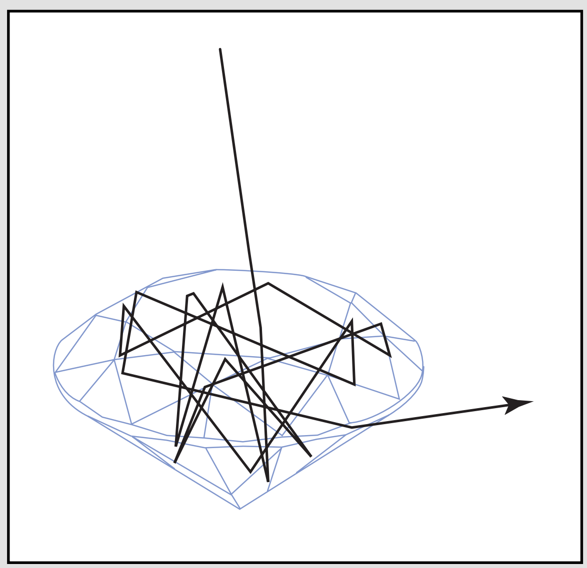
Next we examined mathematically how millions of rays of light from the source interact with the transparent, three-dimensional, colorless, fully faceted round brilliant specified by our choice of proportion parameters. Diamond is a dispersive material; the refractive index is different for different wavelengths of light. Since the angle of refraction depends on the refractive index, white light entering the virtual diamond is spread (dispersed) into rays of different colors, and each of these variously colored rays takes a slightly different path through the stone. We used Sellmeier's formula (see Nassau, 1983 [p. 211]; or, for a more thorough explanation, see Papadopoulos and Anastassakis, 1991) to incorporate this dispersion into the model. With this formula, we obtained the correct refractive index for each of the different colored rays (taken at 1 nm intervals from 360 to 830 nm), so that each ray could be traced (followed) individually as it moved through the stone. Some rays follow simple paths with only a few internal reflections, while others follow complex three-dimensional paths (figure A-2). The color distribution of these rays was scaled to the response of the human eye, using "CIE *X,Y,Z*" color functions as part of the convergence criteria (Wright, 1969).

Figure A-1. We used eight parameters—varied across the range given in table 4—to define our geometric model of the round brilliant shape. (a) All linear distances in this profile view can be described as a percentage of the girdle diameter. The enlarged view of the girdle is centered on the position where we measured the girdle thickness. (b) In this view of the table, the star length *is shown at 50%, so that the star facets extend halfway from the table to the girdle (when viewed from straight above). (c) In this view of the pavilion, the* lower-girdle length *is shown at 75%, so that the lower girdle facets extend three-fourths of the distance from the girdle to the culet (when viewed from straight below).*



Each time a ray strikes a facet, some combination of reflection and refraction takes place, depending on the angle between the ray and the facet, the refractive index at the wavelength of the ray, and the polarization state of the ray. Although the rays from our diffuse hemisphere light source are initially unpolarized, a light ray becomes polarized each time it bounces off a facet. The degree and direction of polarization affect how much of the ray is internally reflected, rather than refracted out the next time it intersects a facet. (For example, about 18% of a light ray approaching a diamond facet from the inside at an angle of 5° from the perpendicular is reflected, regardless of the polarization. But at an incidence of 70°, rays with polarization parallel to the plane of incidence are completely lost from the stone, while 55% of a ray polarized perpendicular to the plane of incidence is reflected back into the stone.) The model traces each ray until 99.95% of its incident energy has exited the diamond. The end result of this ray tracing can be either an image of the virtual diamond or the value of a metric for that stone, or both.

Figure A-2. Some of the light rays that pass through the virtual diamond follow complicated paths. Here, we show the track of one light ray within a round brilliant diamond, as calculated by our mathematical model. This ray reflects in multiple directions. Not all light rays reflect this many times, but most take a three-dimensional path through the diamond. The chief inadequacy of a two-dimensional analysis is that light rays must be confined to a single plane.



20 × 20 × 20 × 20 × 20 × 20 × 20 × 20 = 25.6 billion computations, which was not feasible at this time. (Note that each of these computations would require tracing the previously mentioned 65 billion light rays.)

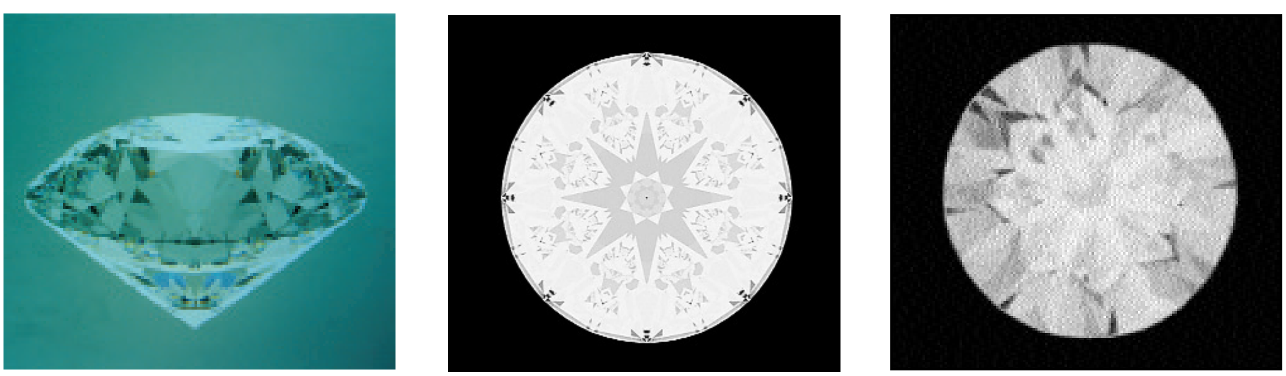
Direct observation of actual diamonds indicates that the overall shape of the round brilliant is primarily determined by three parameters: *crown angle*, *pavilion angle*, and *table size*. (These were the only parameters Tolkowsky considered in his analysis.) Therefore, we examined in detail the changes in WLR as these three important parameters varied together, while holding the others constant. We used about 20 values for each parameter, within the ranges given in table 4. This yielded almost 20,000 proportion combinations, with each calculation requiring several minutes of computer time. We also analyzed the extent to which the other five parameters affect diamond appearance by varying each of them individually while holding the other seven parameters constant at the reference values (again, see table 4).

For each chosen set of cut parameters, our computer program can calculate a single WLR value or an image of the virtual diamond (or both). The WLR values can be compared to one another for different sets of proportions. The bulk of this discussion will focus on the analysis of these WLR values for various ranges of parameters taken singly and in combination. Table 4 lists these ranges for the 20,122 combinations of cut proportions that we have examined for this study. In addition to the WLR values generated for these virtual diamonds, we also examined proportion data obtained from 67,621 actual diamonds measured and graded in the GIA Gem Trade Laboratory (GIA GTL), and we calculated WLR values for virtual diamonds with these combinations of proportions (see Box B).

As a convenience for the readers of this article, for comparison purposes only, we have placed WLR values into five general categories:

High (bright):	WLR values above 0.285
Moderately high:	WLR values from 0.280 to 0.285
Typical:	WLR values from 0.270 to 0.280
Moderately low:	WLR values from 0.265 to 0.270
Low (dark):	WLR values below 0.265

Figure 2. Left: This oblique profile view of a standard round brilliant—a "virtual" diamond (1024 × 1024 pixels)—with commercially acceptable proportions was calculated with our mathematical model using a partial sphere of white light for illumination. This computer-generated image shows how the model captures many of the appearance aspects of an actual diamond, such as three-dimensionality, transparency, facet arrangement, overall light return (brilliance), pattern of light and dark reflections, and dispersion (fire). Center: This digital image (512 × 512 pixels) of a standard round brilliant was calculated face-up with a diffuse hemisphere of white light for illumination. The proportions, used for reference throughout this paper, are: 34° crown angle, 40.5° pavilion angle, 56% table, 3% girdle (at the thickest places, which corresponds to a medium girdle thickness) with 64 girdle facets, 50% star length, 75% lower-girdle length, and 0.5% ("very small") culet. Right: An actual diamond with proportions comparable to the virtual diamond in the center was photographed in diffuse white light using a hemispherical reflector. This diamond has a 34.5° crown angle, 40.9° pavilion angle, 55% table, faceted girdle of medium thickness, 38.7% star length, very small culet, and excellent symmetry. The lower girdle length was not measured. The diffuse illumination reduces the overall contrast, allowing us to examine brilliance separately from the other appearance aspects. Photo by Vincent Cracco.



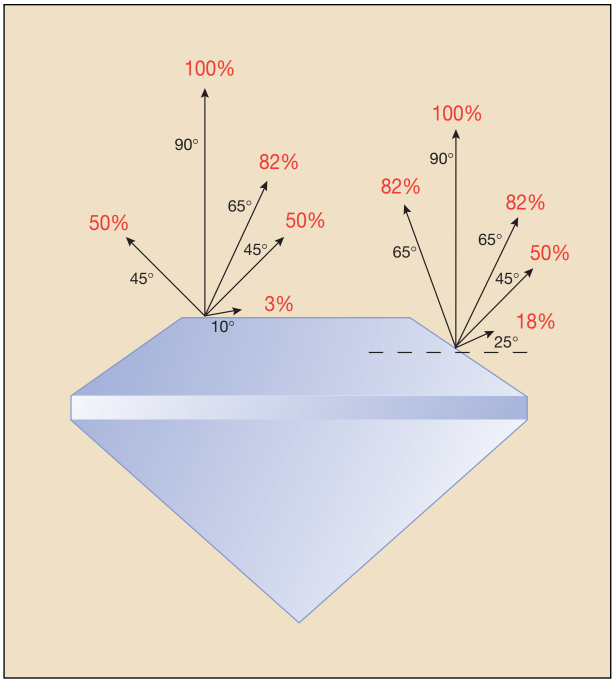


Figure 3. The weighting function used to calculate weighted light return (WLR)—the sum of the squares of the cosines of the angles (from the vertical) at which light rays emerge from the diamond—depends on the angle of the emerging ray to the table plane, regardless of which crown facet it exits. As the relative lengths of the arrows on this illustration indicate, light rays that emerge straight up (perpendicular to the table) are given 100% weight, rays that exit at 45° to the table are given 50% weight, and the rays that exit at 0° (parallel to the girdle plane) are given zero weight (i.e., they do not contribute to the total WLR).

These groups should *not* be taken as WLR or brilliance "grades." The authors strongly caution against such usage. These terms are provided as a convenience only, to compare the relative brightness of the virtual diamonds in the different WLR ranges.

As seen in figure 1, large differences in WLR correlate to perceptible differences in the overall brightness of actual diamonds. Even over the restricted WLR range in the inset to figure 1, the darkest and brightest stones differ by almost 0.010; this difference is also easily perceived by a trained viewer. In our experience, WLR differences of 0.005 are discernable among stones with the same color and clarity grades when examined with controlled observation environments and lighting conditions.

RESULTS

Images and WLR. The calculations made with our model produced realistic digital images of virtual diamonds (again, see figure 2). These computer-generated images reproduce the patterns of light and dark seen in actual round brilliant diamonds under lighting conditions similar to those used with the model, although the light-and-dark patterns are more symmetrical than those seen in most real diamonds. During the course of this research, we generated a variety of digital images, from different perspectives and with different lighting conditions. However, the details of how brilliance changes with proportions can be better studied by comparing a metric, such as WLR values, than by visually examining thousands of images.

Results for Key Individual Parameters. Our investigation of the dependence of WLR on crown angle, pavilion angle, and table size began with an examination of how WLR varies with each of these three parameters while the remaining seven parameters (again, see Box A) are held constant. Except where otherwise noted in the text, we fixed these parameters at the reference proportions that are provided in table 4.

TABLE 4. The eight proportion parameters used for calculating the WLR values.

Round-brilliant-cut parameters[a]	Range[b]	Increment	Reference proportions
Table size[c]	50% – 75%	1%	56%
Crown angle[c]	19° – 50°	1°	34°
Pavilion angle[c]	38° – 43°	0.25°	40.5°
Girdle facets[d]	16 – 144	16	64
Girdle thickness[d]	1.8% – 4.0%	0.2%	3.0%
Star length[d]	5% – 95%	1%	50%
Lower-girdle length[d]	50% – 95%	5%	75%
Culet size[d]	0% – 20%	0.5%	0.5%

[a] *These eight parameters are defined in Box A.*
[b] *These ranges extend beyond the widest range for diamonds normally encountered in the trade today.*
[c] *These three parameters were varied together while the other five were held at the reference proportions. A set of calculations was also made at the reference proportions with crown angle varying from 1° to 50°.*
[d] *These five parameters were varied individually. For each one, the other seven parameters were held constant at the reference proportions.*

BOX B: COMPARISON OF MODELED RESULTS TO ACTUAL DIAMOND PROPORTIONS

To get an idea of the range of WLR values for stones seen in the diamond trade, we collected information on the proportions of 67,621 round brilliants graded by GIA GTL. (This data set included all the D-to-Z-color round brilliant diamonds seen during a span of months.) This population of diamonds had crown angles ranging from 19.4° to 45.0°, pavilion angles from 39.9° to 43°, and table sizes from 50% to 74%. More than 80% of this group of diamonds fell within the smaller proportion range of 30°–40° crown angles, 40.2°–42.4° pavilion angles, and table sizes from 53% to 70%. The WLR values calculated for all of the diamonds ranged from 0.235 to 0.306, with a mean and standard deviation of 0.274 ± 0.005. The stones with average proportions for this sample set (represented by 29% of the sample) had crown angles between 31° and 35.9°, pavilion angles between 41.0° and 42.4°, and table sizes of 61%–63%; the WLR values calculated for this relatively small range of proportions varied from 0.269 to 0.279.

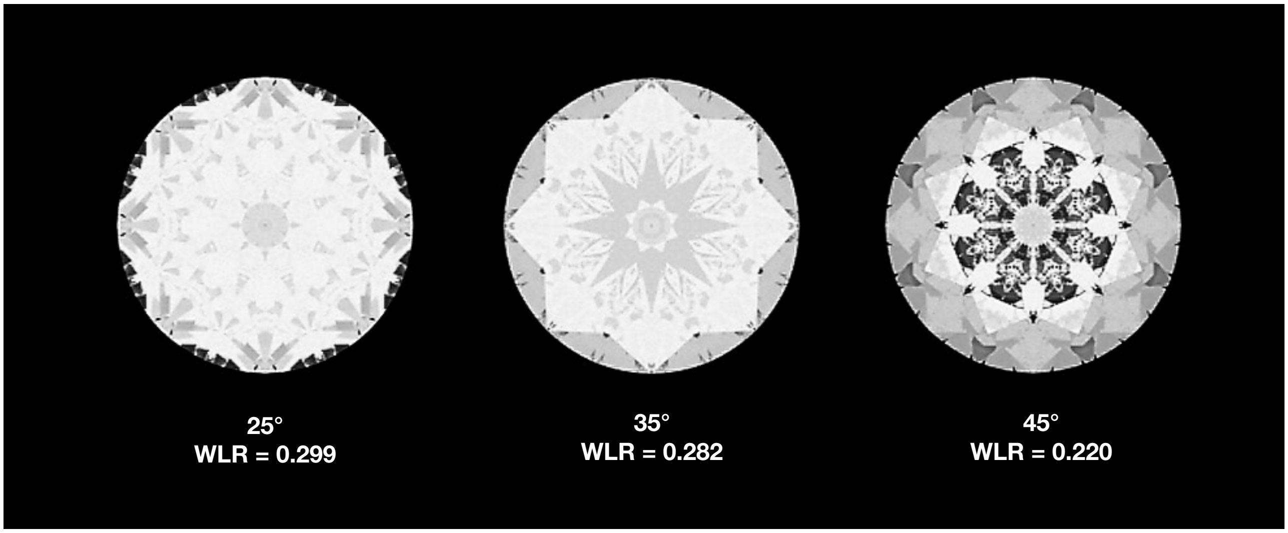
In the entire data set, the diamonds with the highest calculated WLR values had the smallest crown angles: only eight of the 67,621 stones had WLR values above 0.295 (far into the high range), and of these, the largest crown angle was 25.5°. However, crown angle alone does not determine WLR; the 61 stones with crown angles less than 25° had WLR values ranging from 0.261 to 0.306 (low to high), with an average of 0.288 (high). Another 3,494 stones had crown angles between 25° and 30°, with more than half of these falling between 29.0° and 29.9°, and WLR values from 0.261 to 0.296 (low to high). In contrast, round brilliants with high crown angles have lower WLR values on average, although the brightest such stones yield WLR values slightly higher than the mean for the whole population; 7,617 diamonds had crown angles of 36° or more, with WLR values that ranged from 0.235 to 0.278 (low to typical). There were 275 round brilliants that had crown angles of 40° or more, with WLR values ranging from 0.235 to 0.259 (all low); these values indicate diamonds that are considerably darker than the average.

This sample of 67,621 diamonds contained very few with proportions that would qualify for a top grade in most of the systems shown in table 3. Only 3% of the sample (2,051 round brilliants) had crown angles between 34.0° and 34.9°, pavilion angles between 40.2° and 41.3°, and table sizes between 53% and 57%.

Of these 2,051 round brilliants, only 76, or less than 4% of this group, had tables of 53%, and nearly twice as many diamonds had pavilion angles of 41°–41.3°, rather than 40.2°–40.9°. Thus, even manufacturers who strive to cut "Ideal" proportions often choose to cut a larger table or steeper pavilion angle than Tolkowsky recommended, presumably for greater weight retention. However, there is as yet no clear evidence whether either of these changes significantly alters the balance between brilliance and fire that Tolkowsky proposed. As shown in table 3, the proportion ranges that define the top grades in existing systems yield WLR values of 0.275–0.285 (typical to moderately high); yet some proportions that receive lower grades in thse same systems have higher calculated WLR values.

Crown Angle. Of the three parameters, changing the crown angle produced the greatest variation in WLR. In general, WLR increases as crown angle decreases; but, as figure 4 shows, there are three local maxima in WLR across the range of angles (that is, WLR varies up and down with changes in crown angle). These results suggest that, at the reference proportions, a diamond with a 23° crown angle is brighter than a stone with any other crown angle greater than 10°. However, other considerations may dictate that a diamond must have a crown angle above a certain value (such as reduced durability with a thin to medium girdle and a crown flatter than, say, 25°; see, e.g., Crowningshield and Moses, 1997). Ironically, the highest WLR values are obtained for a round brilliant with no crown at all. It is interesting to note that the question of reduced durability with a shallower crown was discussed in *Gems & Gemology* in 1936, although at that time it was the "modern" trend toward angles of 34.5° (from the steeper angles cut previously) that raised concern (Ware, 1936).

Figure 4 also shows images of virtual diamonds that have crown angles of 25°, 35°, and 45°, with all other parameters at the reference proportions listed in table 4. The overall brightness clearly decreases



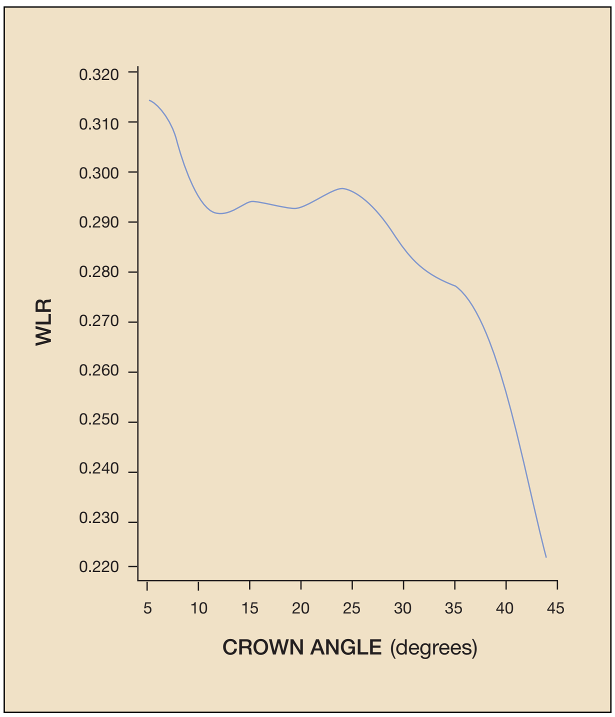


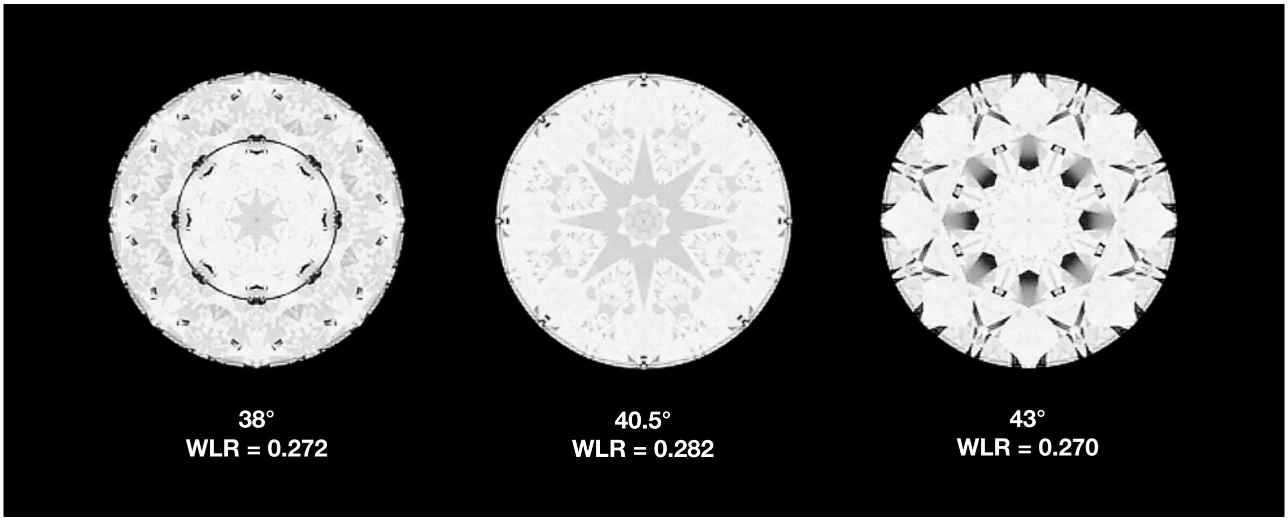
Figure 4. These digital images show how variations in the crown angle from 25° to 45° affect the appearance of a standard round brilliant diamond. All other proportions are held constant at the "reference proportions" shown in table 4. These digital images confirm what the graph of crown angle and WLR illustrates: WLR is highest at very shallow crown angles. The graph shows local maxima at 13°, 23°, and 34°, it begins to drop sharply at crown angles above 38°.

as crown angle increases, but the pattern of light and dark also changes substantially.

Pavilion Angle. This is often cited by diamond manufacturers as the parameter that matters most in terms of brilliance (e.g., G. Kaplan, pers. comm., 1998). With all other parameters at the reference positions, we see a smooth decrease in WLR away from a maximum at about 40.7° (figure 5). Images of virtual diamonds with low, optimal, and high pavilion angles (again, see figure 5) are consistent with the appearances that we would expect for actual diamonds with these pavilion angles ("fish-eye," normal, and "nail head," respectively; see *GIA Jeweler's Manual* [1989]). However, note that although the pavilion angle is optimal at 40.7° when the other parameters are at the reference values, this need not be the case in general. For instance, we calculated the WLR values of a diamond with a completely flat crown. As the pavilion angle of this "crownless" virtual diamond increased from 38° to 43°, WLR increased smoothly from 0.270 to 0.340.

Table Size. With other proportions held at the reference parameters (again, see table 4), there is one broad maximum for WLR as a function of table size, as shown in figure 6. This maximum extends from about 53% to 59%; it is followed by a gradual decrease as table size increases to 70%. (A small shoulder is seen between 72% and 73%.) However, WLR as a function of table size behaves quite differently when this parameter is varied together with crown angle and pavilion angle, as discussed below.

Combined Effects. Some of the interactions between crown angle, pavilion angle, and table



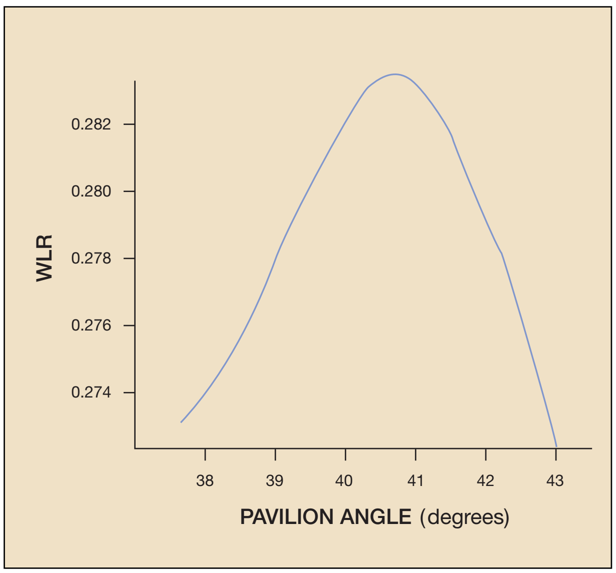


Figure 5. Variations in pavilion angle also affect the appearance of a faceted diamond. These virtual diamonds have pavilion angles of 38°, 40.5°, and 43°; all other parameters are set to the reference proportions (table 4). Although diffuse illumination reduces the contrast in all three images, they do illustrate well-known optical effects, including the "fish-eye" that results from a very shallow pavilion (far left), and the "nail head" caused by a very deep pavilion (far right). The graph of WLR as a function of pavilion angle (with all other parameters at reference proportions) shows a maximum at a pavilion angle of 40.7°.

size—and their joint effects on WLR values—can be seen when these proportion parameters are examined two at a time. One way to visualize these effects is to draw them to look like a topographic map (which shows the differences in elevation of an area of land). We can draw subsets of the data as cross-sections (slices) through the data set with one parameter held constant, and the WLR values can then be expressed as contours. These cross-sections can be read in the same manner as topographic maps; but instead of mountains, these "peaks" show proportion combinations that produce the highest calculated WLR values for diamonds within a small range of proportions.

As illustrated in figure 7, when the crown angle and table size are varied together, the WLR changes in an unexpected fashion. There are "ridges" at crown angles of 23° and 34°. Along these ridges, round brilliants with large tables show unexpectedly high WLR values: For instance, for a 40.5° pavilion angle, a virtual diamond with a 65% table and a 23° crown angle returns more light (high WLR of 0.288) than one with a 56% table and a 34° crown (moderately high WLR of 0.283; again, see figure 7). Although the first of these stones is not a typical commercial cut, crown angles this low are sometimes seen at GIA GTL. In addition, at crown angles up to 37°, the table size has a significant influence on WLR; in general, WLR increases as table size decreases within this range.

When we attempt to illustrate the effects of all three parameters at the same time, the limitations of graphing on two-dimensional paper are readily apparent. The projection of a "three-dimensional graph" in figure 8, for example, shows contours of constant WLR against crown angle, pavilion angle, and table size. This figure shows clearly that the higher-value WLR surfaces have a complex dependence on the combination of these three parameters. In particular, the white contour (WLR of 0.275–0.280) is concave as well as convex, and defines a broad range of proportions that have the same WLR values. However, only a limited region of the WLR surfaces can be displayed on such a graph.

To better show this complexity, figures 9–11 illustrate the results for proportion combinations from three perspectives: constant table size, constant pavilion angle, and constant crown angle. Three points representing distinct sets of proportions are plotted on these cross-sections; the point labeled A, for example, shows the position of a virtual diamond with the "reference proportions" (again, see table 4) in each of the three perspectives. Tolkowsky's proportions are shown as point T. Point B represents another high-WLR virtual diamond with a shallower crown angle.

Constant Table Size. Figure 9 shows the "topography" of the WLR values in a series of slices through surfaces of constant table size. It provides data for virtual diamonds with crown angles between 28.5° and 37.5°, and pavilion angles between 38° and 43°, at table sizes ranging from 50% to 66%. Overall, WLR is highest for fairly small tables (53% to 59%), and increases as crown angle decreases. Note the ridge of higher WLR that trends from the lower left corner of each constant-table-size slice to the center of the right side. This ridge becomes broader and shallower (smaller range of WLR values) as table size increases. It is evident that this complexity cannot be accounted for in a model of diamond proportions that treats the optimal set as the center of a three-dimensional "bull's-eye" pattern, surrounded by concentric shells of worsening results.

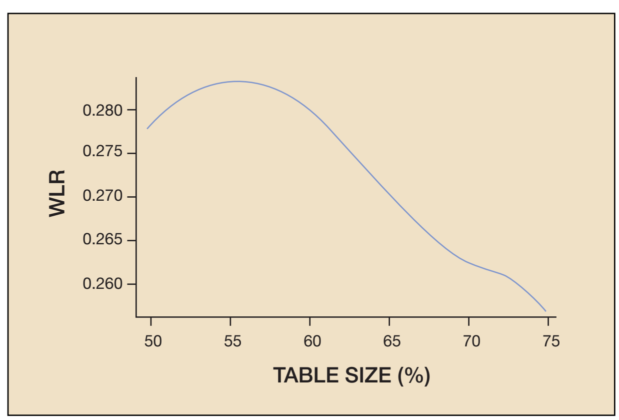
Constant Pavilion Angle. In figure 10, we show slices through the data at constant pavilion angle. The complex nature of the WLR surfaces is apparent from this view as well. The cross-section for a 39.3° pavilion angle shows a pronounced ridge of higher WLR values starting in the upper left corner (shallow crowns and small tables), and trending toward higher crown angles at table sizes less than 63%. This ridge is seen at all higher pavilion angles as well; it is the same ridge seen at higher crown angles in figure 7, as viewed from many different pavilion angles.

As the pavilion angle increases, the ridge defined by the WLR values seen in the orange, yellow, and white areas of figure 10 extends first to higher crown angles, and then broadens to include larger tables. From this perspective, it is clear that pavilion angle can interact strongly with the other two proportion parameters to produce similar WLR values across broad ranges of crown angle and table size.

Constant Crown Angle. From the third perspective, constant crown angle (figure 11), the WLR contours look much smoother. They form broad oval curves at shallow crown angles, with oval maximum regions at crown angles between 30.5° and 36.5°, surrounded by relatively smooth contours of decreasing WLR.

For crown angles greater than 30°, the set of optimal parameters appears in this perspective as a "bull's-eye" pattern, where a deviation in any direction worsens the WLR. However, the pavilion and table slices demonstrate that—in terms of WLR—there are many proportion combinations that yield equally bright results.

Figure 6. This graph of WLR versus table size, with all other proportions at the reference values, shows a broad maximum centered at about 56% and a gradual drop-off toward both smaller and larger table sizes. The wide, gently sloping top of this graph suggests that WLR is not strongly affected as table size varies between 50% and 62%, for these properties.



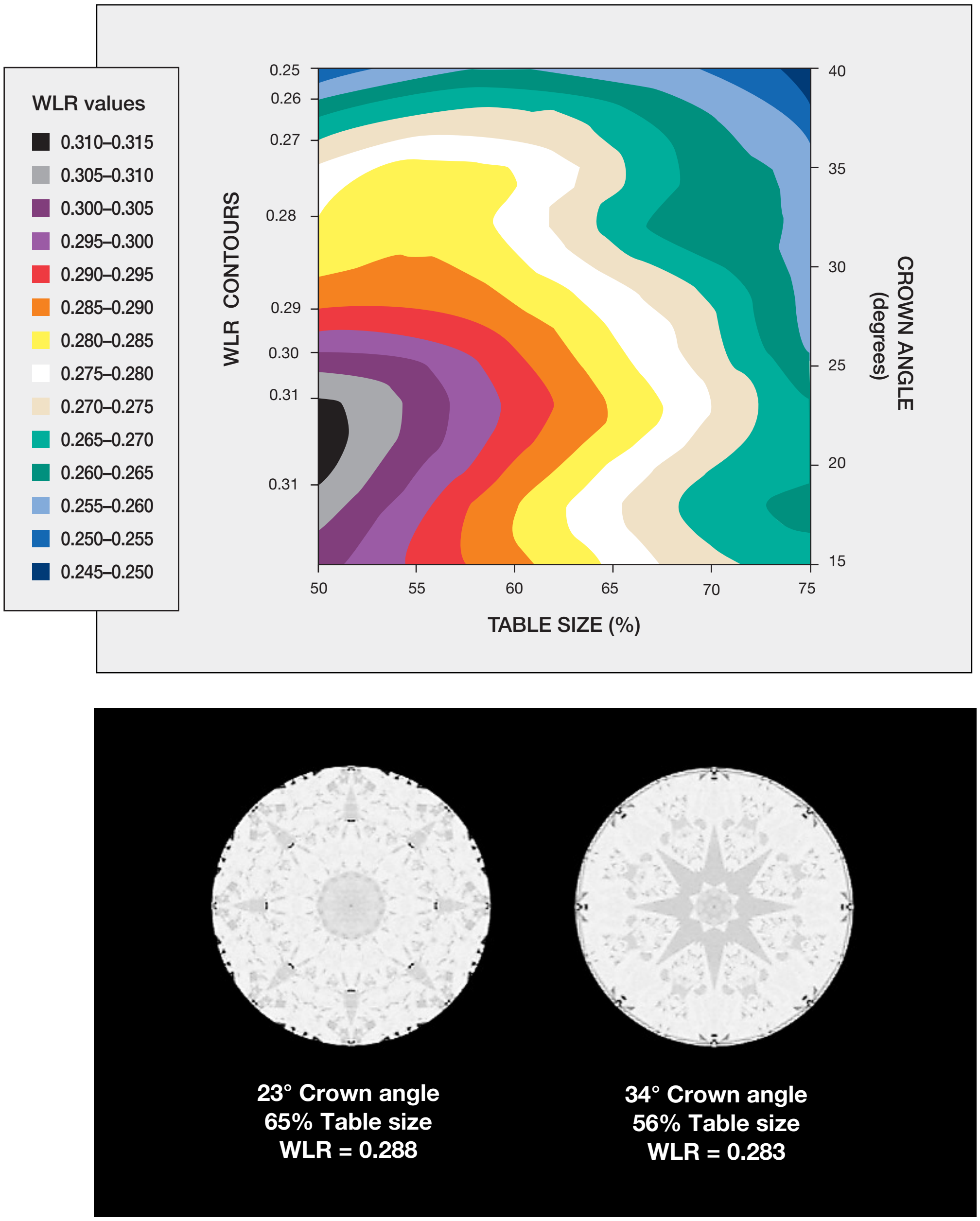


Figure 7. This diagram shows the variation in WLR with changes in both crown angle and table size. The WLR values are contoured, like a topographic map; each line is a constant WLR value. Regions with the highest WLR are shown in black, gray, and purple. High WLR values are found at crown angles up to 35°, beyond which WLR decreases rapidly. Note that the highest WLR values are seen at small table sizes. The shape of the 0.280 WLR contour line reflects the variation in WLR due to different crown angles; the local maxima at 23° and 34° in figure 4 become "ridges" in this figure. Note also the prominence of the "ridge" at the 23° crown angle; WLR remains high for much larger tables than at either larger or smaller crown angles. The digital images show how the pattern of light and dark can differ for proportions leading to similar WLR values. The virtual diamond on the left has a 23° crown angle and a 65% table (high WLR of 0.285), and the one on the right has a 34° crown angle and a 56% table (moderately high WLR of 0.283); all other proportions for both virtual diamonds were held constant at the reference proportions in table 4.

Influence of the Other Five Parameters on WLR Results. We explored the contribution to WLR of the remaining five proportion parameters—girdle thickness, number of girdle facets, culet size, lower-girdle length, and star length—by calculating WLR values while varying one parameter and holding the other seven (including crown angle, pavilion angle, and table size) constant at the reference proportions in table 4. We found that WLR decreases slightly as the thickness of the faceted girdle increases, presumably because more light is lost through a thicker girdle. In addition, WLR was approximately constant as the number of girdle facets varied from 32 to 144. A smaller WLR value was obtained for the extremely low value of 16 girdle facets.

We expected to see a steady decrease in WLR with culet size, similar to that seen for girdle thickness; instead, we found relatively constant values for culets as large as 12% (which would be described as very large), and then a decrease as the culet size increased further. Although we have not yet examined this result in significant detail, it implies that relatively few light rays escape through the culet.

The graph of WLR versus lower-girdle length shows a smooth curve with one maximum, similar to the curve seen in figure 5. With all other proportions at the reference values, we found the highest WLR when the lower-girdle length was 70%, rather than the commonly cut 75%. The total variation in WLR was small, and optimization of this parameter showed a strong dependence on star length.

The graph of WLR versus star length shows several local maxima (figure 12A). The overall maximum is found for a star length of 57%, rather than the 50% that is commonly cut for round brilliants. However, WLR varies by only 0.010—from 0.274 (typical) to 0.284 (moderately high)—over the range of 25% to 95% star length. With the crown angle fixed at the reference value of 34°, the 57% star length corresponds to a star facet angle of 22.5° and an upper-girdle-facet angle of 41.2° (WLR of 0.284), while the 50% star length yields a star facet angle of 21.5° and an upper-girdle-facet angle of 40.4° (WLR of 0.282; see figure 12B). This change makes a rather subtle difference in the profile of the diamond, producing a slightly steeper profile along these two facets without any change in crown height. Although WLR varies only a little, indicating little change in brilliance, the pattern of light and dark across the crown changes significantly, as shown in the digital images (figure 12C).

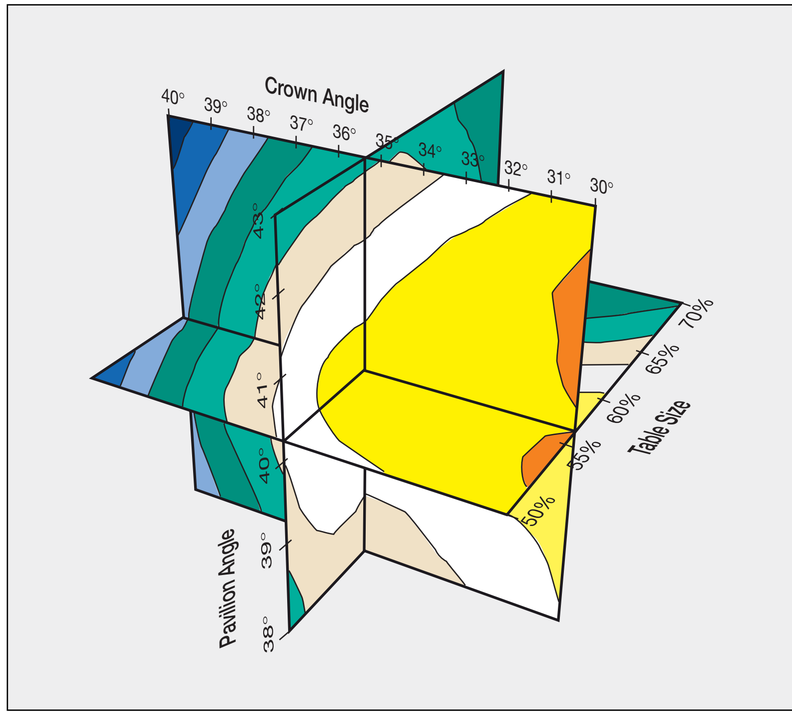


Figure 8. The full complexity of the WLR surfaces becomes apparent when we vary all three parameters—crown angle, pavilion angle, and table size. The contours show constant values of WLR in increments of 0.005, from above 0.285 for the orange area to below 0.250 for the dark blue area. The greatest complexity in the contours is seen at the highest WLR values. However, since this three-dimensional projection is drawn from only one perspective, it cannot show all the variations in the WLR surfaces.

DISCUSSION

Verification of the Model. To verify our study, we need to ask whether our model adequately reproduces both the visual appearance of white light return from actual diamonds and the effects of cut that are familiar from observation of actual diamonds. The data indicate several similarities in appearance between the virtual diamonds generated with this model and actual stones. As we saw in figure 5, the virtual diamond images showed characteristics of actual faceted diamonds (e.g., "fish-eye" and "nail head" appearances), as pavilion angle alone was changed. Similarly, we found a sharp decrease in WLR for crown angles above 38°, and actual stones with such steep crown angles may look darker (see, e.g., figure 1).

The most meaningful test of our mathematical model is to compare the calculated WLR values to the appearance of actual diamonds with those same proportions. Figure 1 shows photos of actual diamonds with proportions that correspond to varying WLR values. As table 1 indicates, the stones in figure 1 have proportions that would fall in four of the five general categories of WLR values: (1) high (calculated WLR greater than 0.285); (2) moderately high, which includes the proportion ranges of many professed "superior" cuts (from Tolkowsky, Eppler, and Eulitz in table 2; WLR range of 0.280–0.285); (3) typical (WLR range of 0.270–0.280); and (4) low (WLR less than 0.265). However, because WLR measures light return from many different perspectives, not just one, no single photograph can demonstrate WLR results exactly.

Using WLR Data to Evaluate Brilliance. The WLR surfaces that we have calculated as a function of crown angle, pavilion angle, and table size are irregular, with a number of maxima, rather than a single maximum. These multiple "peaks" are a principal result of this extensive three-dimensional analysis. Their existence supports a position taken by many

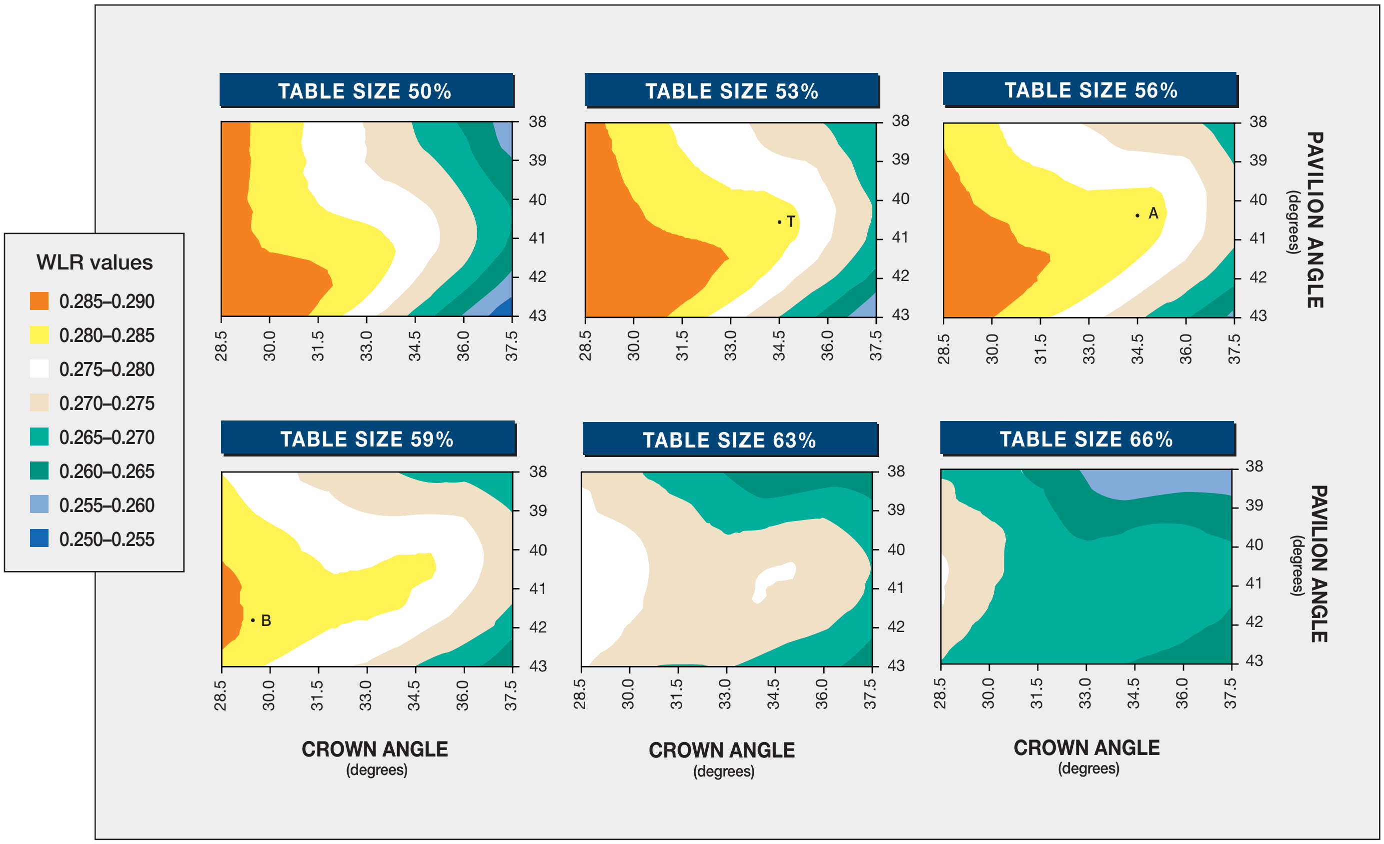


Figure 9. A series of slices through the data plotted in figure 8 makes it easier to see how WLR changes as these three parameters vary. Each plot shows contours of constant WLR for a given table size, as crown angle varies along the horizontal axis and pavilion angle varies along the vertical axis. Note that the contours define irregular surfaces. In general, the WLR values increase as table size decreases, with the highest values at a table size of 53% (for 30° and higher crown angles). The WLR values are also higher at intermediate pavilion angles and at lower crown angles. Three points are marked on these plots: Point A denotes a virtual diamond with a 34.5° crown angle, a 40.7° pavilion angle, and a 56% table (our reference proportions), with a WLR value of 0.282. Point B shows the location of a virtual diamond with a 29.5° crown angle, a 41.7° pavilion angle, and a 59% table, with a WLR value of 0.284, and point T marks a virtual diamond with a 34.5° crown angle, a 40.7° pavilion angle, and a 53% table (Tolkowsky's proportions), which yields a WLR value of 0.281. These same three points are shown in figures 10 and 11 as an orientation aid: Each point marks the same set of proportions.

in the trade (see, e.g., Federman, 1997): At least in terms of light return, or "brilliance," there are many combinations of parameters that yield equally "attractive" round brilliant diamonds. This interaction between the proportion parameters is not taken into account by existing cut-grading systems, which examine each parameter separately.

It is especially important to note that some proportion combinations that yield high WLR values are separated from one another and not contiguous, as shown in the cross-sections of the WLR surfaces. Thus, for some given values of two proportions, changes in the third proportion in a single direction may first worsen WLR and then improve it again. This variation in WLR with different proportion combinations makes the characterization of the "best" diamonds, in terms of brightness, a great challenge. Even for one simple shape—the round brilliant cut—and variation of only three proportion parameters, the surfaces of constant WLR are highly complex.

The specific proportion combinations that produce high WLR values have a variety of implications for diamond manufacturing. Because many combinations of proportions yield similarly high WLR values, diamonds can be cut to many choices of proportions with the same light return, which suggests a better utilization of rough (see Box C).

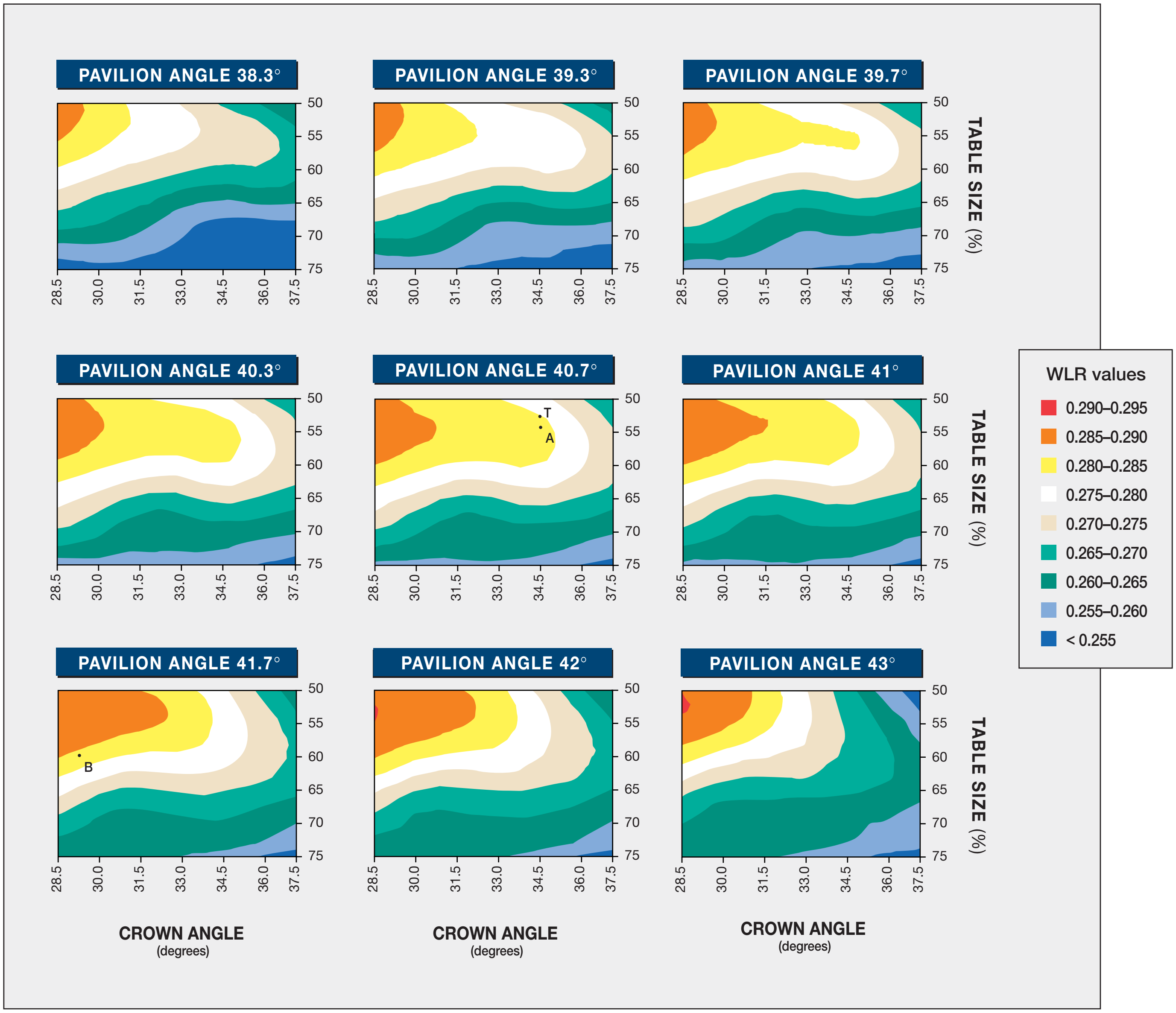


Figure 10. Each plot in this figure shows contours of constant WLR for a given pavilion angle, as crown angle varies along the horizontal axis and table size varies along the vertical axis. Here again, the complex nature of the WLR surfaces is apparent in the patterns shown in these cross-sections. The highest WLR values are seen at higher pavilion angles, for very shallow crown angles and small tables. In general, higher WLR values are found for the widest range of crown angles and tables sizes as the pavilion angle tends toward 41°. Points A, B, and T from figure 9 are shown as orientation aids.

Evaluation of "Superior" Proportions Suggested by Earlier Researchers. Because a gem diamond should display an optimal combination of brilliance, fire, and pleasing scintillation, the best overall appearance might not be found among the brightest round brilliant cuts. According to our WLR calculations, however, some of the "superior" proportions proposed by other researchers (see, e.g., table 2) do not produce a reasonably bright diamond—for example, those from Stoephasius (1931; especially the one with a 43.8° crown angle, with a calculated WLR of 0.216) and Suzuki's Dispersion Design (1970; even Suzuki's Brilliance Design, with a WLR of 0.252, calculates as dark). Tolkowsky's proportions yield a moderately high WLR of 0.281. It is interesting to note that only seven of the 31 sets of superior pro-

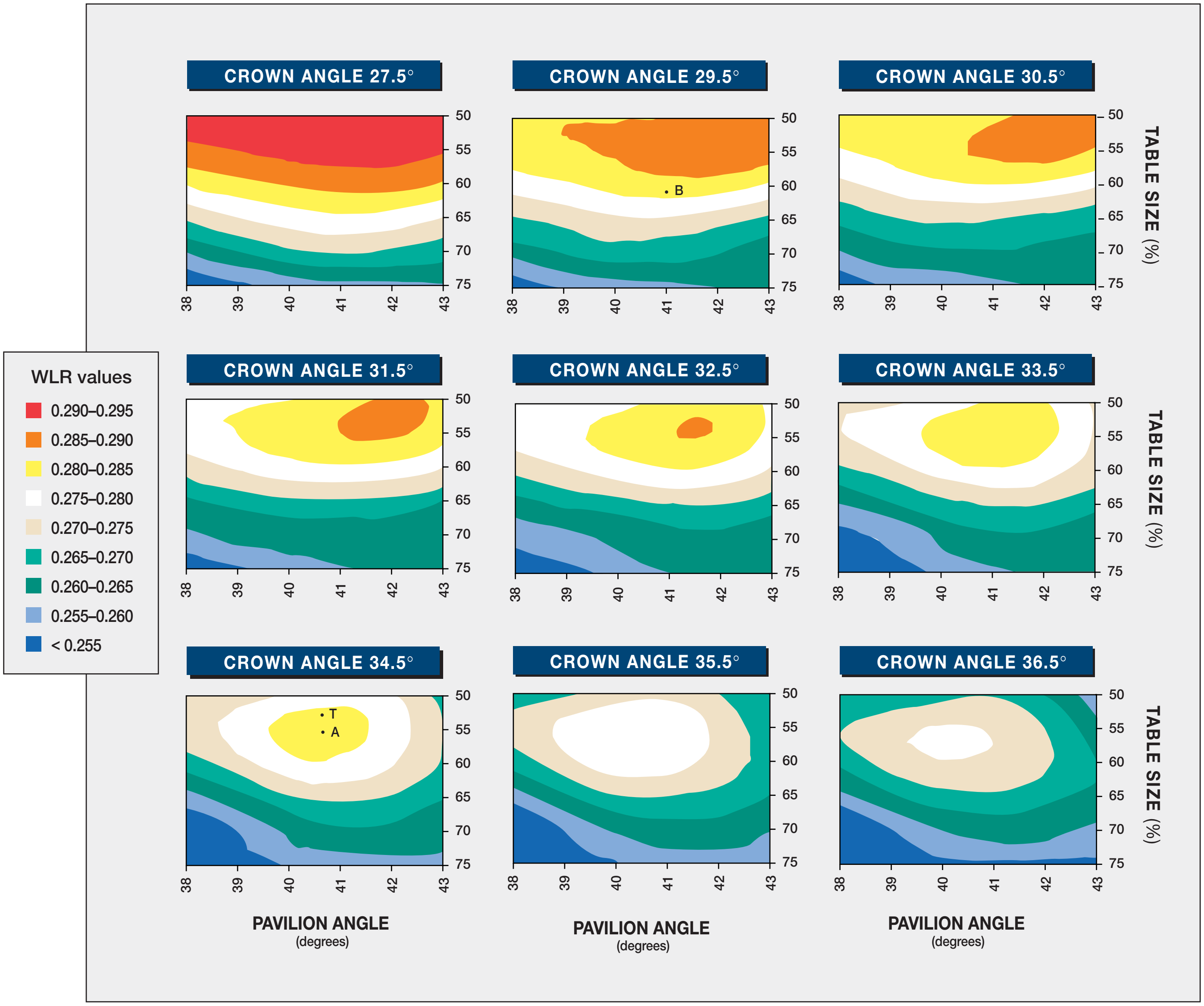


Figure 11. Each of the slices in this set shows contours of constant WLR for a given crown angle, as table size varies along the vertical axis and pavilion angle varies along the horizontal axis. WLR surfaces look much smoother and more concentric from this view, and generally WLR decreases as crown angle increases. Points A, B, and T from figure 9 are shown as orientation aids.

portions proposed since Tolkowsky have better calculated WLR values (Eppler, 1939; Parker, 1951 [cited by Eppler, 1973]; Schlossmacher, 1969; Eulitz, 1972; Scandinavian Diamond Nomenclature Committee, 1979; Dodson "most fire," 1979; and [one of four] Shannon and Wilson [Shor, 1998]). Relative to Tolkowsky's proportions, all of these have larger tables (56%–60%) and shallower crowns (25.5°–33.6°); all but one have comparable pavilion angles (40.7°–40.9°; the exception, Dodson's "most fire," has a 43° pavilion angle). The highest WLR (0.297) is calculated for Parker's (1951) cut, with a 55.9% table, 25.5° crown angle, and 40.9° pavilion angle.

Recent work by Shannon and Wilson, as described in the trade press (Shor, 1998), presented four sets of proportions that they claimed gave "outstanding performance" in terms of their appearance. Yet we calculated typical to moderately high WLR values for these proportions (again, see table 2). In comparison, Dodson's (1979) proportions for the "most brilliant" diamond yield a WLR value of

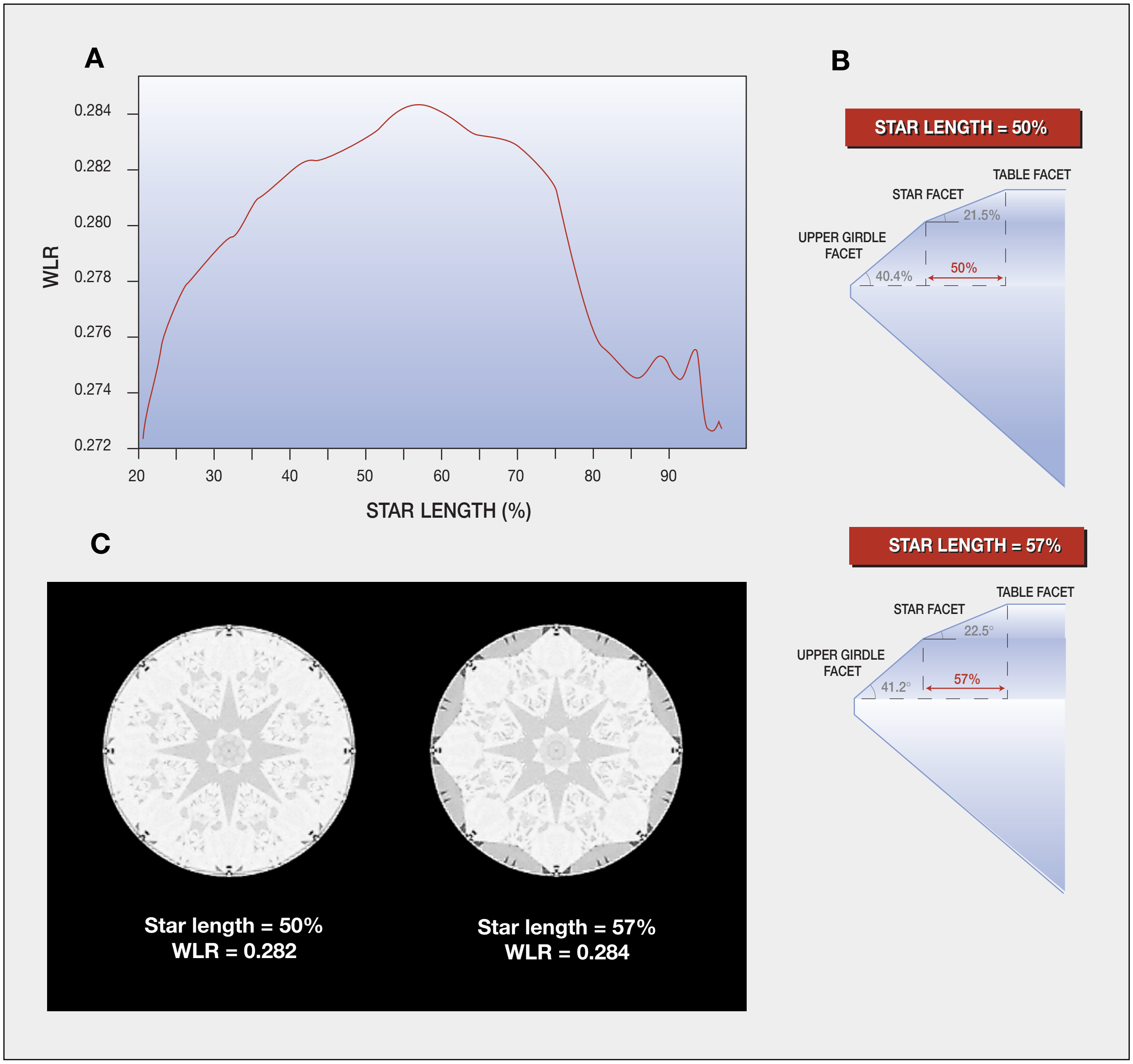


Figure 12. (A) The graph of WLR versus star length (with all other parameters held constant at the reference proportions) shows many local maxima within a relatively small range of WLR. This calculated WLR implies that brilliance can be increased slightly if the star length is increased from the usual to 57%. (B) These diagrams show how longer star length results in slightly steeper angles for both the upper girdle facets and the star facets. The upper diagram, with a star length of 50%, corresponds to the reference proportions in table 4; the lower diagram shows a star length of 57%. (C) The virtual diamond images are of diamonds with a 50% star length (left) and a 57% star length (right). Although the image on the right is darker around the edge, it has a slightly higher WLR value (0.284) than the image on the left (0.282).

0.277 (typical range) for a 40% table size (much smaller than any commercially cut stones), a 26.5° crown angle, and a 43° pavilion angle. However, Dodson also evaluated one metric each for fire and "sparkliness" for four table sizes, three crown heights, and 10 pavilion angles. His "most fire" proportions gave a high WLR of 0.287, which is far brighter by our calculations than his "most brilliant" stone. The differences between our weighting technique and those of Dodson and of Shannon and Wilson are probably responsible for these discrepancies.

Implications for Existing Cut-Grading Systems. Our results disagree with the concepts on which the proportion grading systems currently in use by various laboratories appear to be based. In particular, they do not support the idea that *all* deviations from a narrow range of crown angles and table sizes should be given a lower grade. We have calculated the WLR values for the proportion ranges of each grading system in table 3. The highest grades for most of those systems yield WLR values from 0.275 to 0.284 (typical to moderately high). Clearly, these are attractive stones. However, the maximum WLR achievable increases as the grade worsens in these systems.

For example, diamonds with a 31° to 32° crown angle, a 41° to 41.4° pavilion angle, and a table size between 53% and 57% have WLR values of 0.284–0.285 (moderately high). Although their WLR values are slightly higher than those of the top grades in table 3, these round brilliants would receive lower cut grades in most systems because of the lower crown angle. Similarly, diamonds with crown angles between 31° and 33°, pavilion angles of 42°, and tables between 53% and 59% yield calculated WLR values from 0.281 to 0.286 (moderately high to high). These values may exceed those of diamonds that currently receive the best grades, but such stones earn medium to low grades from the existing systems because of the larger pavilion angle. Last, round brilliants with larger tables (61% to 63%) are much more common than those with small tables (again, see Box B). Such diamonds can show moderately high WLR values when combined with crown angles between 30° and 33°, and pavilion angles from 40° to 42°, but diamonds with large tables are penalized heavily in most of the existing cut grading systems, regardless of their brightness.

Although arguments can be made for downgrading diamonds with lower crown angles or larger tables (on the basis, for example, that they do not yield enough fire), there is little documented evidence at present to support—or refute—such claims. However, at least according to Dodson (1979), both fire and scintillation depend on combinations of proportions, rather than on any single parameter.

Although our results for brilliance do not support current cut grading systems, we do not expect them to surprise most diamond manufacturers. GIA GTL has seen significant numbers of diamonds that are cut to various proportion combinations that would correspond to moderately high to high WLR values. The results of this study support the empirical understanding that cutters have of the relationships between proportions and brilliance.

FUTURE DIRECTIONS

The model presented here can be used readily to explore many aspects of how diamond cut affects appearance. The greatest challenge in this research is the derivation of metrics for appearance concepts, including selection of the best lighting and observation conditions for each metric. Currently, we are exploring metrics for fire, which has many possible variables, such as: the size, extent, placement, and exit angle of colored light rays; the distribution of colors observed; and how the observation and lighting geometries govern the recombination of colored light rays into white light. We plan to devise a metric for scintillation as well, and to compare these results over the same proportion ranges to the metrics for brilliance and fire. We also intend to explore other lighting conditions, as we develop metrics for the other appearance concepts.

In addition, we plan to explore two important considerations that have been neglected thus far: symmetry and color. From our efforts and observations of actual diamonds for this study, we suspect that symmetry deviations may produce significant variation in brilliance (this was also suggested by A. Gilbertson, pers. comm., 1998). Incorporation of symmetry deviations requires adding more parameters to describe the shape of the round brilliant, and devising a method of tracking multiple symmetry faults. Once this is done, the model can be used to calculate both images and metric values that show how symmetry deviations, both singly and in combination, change diamond appearance.

Incorporating color, whether letter grades (e.g., from J to Z) or fancy colors, requires giving the virtual diamond a set of dimensions, applying a specific absorption spectrum, and specifying the color distribution (even or zoned). Then the model can keep track of the energy a ray loses by absorption (in addition to leakage) as it travels through the virtual diamond. Fluorescence effects can be included by similar techniques (applying a fluorescence spectrum), and the claim that fluorescent diamonds look better at different proportions than those that are inert (G. Tolkowsky, 1996) can be directly evaluated.

This model can also be used to explore the many

BOX C: AN EXAMPLE ILLUSTRATING CUTTING CHOICES

Fully symmetrical octahedral rough lends itself quite nicely to the higher crown and smaller table typical of the "Ideal" cut, as shown in figure C-1. However, several diamond manufacturers have estimated that only about one fourth of the rough they cut is fully symmetrical. (A. D. Klein, pers. comm., 1998). Other rough shows some irregularity in shape: shorter along one point-to-point distance than the other two; one or more flattened edges with minor development of cube or dodecahedral faces; or some relative tilt or twist between the two pyramids that comprise the octahedron. These variations in the shape of the rough can be accommodated during cutting either by accepting a lower weight yield or by modifying the cutting proportions.

If we consider a typical slightly asymmetric octahedron (see, e.g., figure C-2), one could still work toward an "Ideal" cut despite the limitations of the rough. Choosing to saw such a piece just slightly off center separates the top from the bottom of the octahedron, yielding two symmetrical square pyramids; for the purposes of this example, let us assume that the larger of these two pieces weighs about 1.75 ct. Aiming for a crown height of 16% or 17% (which allows for a crown angle of 34°–35°, at a range of table sizes) pushes the girdle down below the widest part of the rough, forcing a lower yield. After exploring the possibilities for this example with a DiaExpert system (see, e.g., Caspi, 1997), the best yield we found for top-graded proportions as defined by most of the cut-grading systems (see table 3), was 0.93 ct, with a 35.5° crown angle, a 40.8° pavilion angle, and a 57% table, giving a calculated WLR value of 0.279.

However, the shape of this rough suggests a different sawing position; it promotes cutting a shallower crown and a larger table. From the same sawn bottom piece of about 1.75 ct, one could plan a round brilliant with a 60% table and about a 60% total depth, with a 32.7° crown angle and a 41.5° pavilion angle, which would achieve a calculated WLR value of 0.279 and a final weight of 1.02 ct. In this example, striving for a high cut grade (table 3) results in a substantially lower weight yield while achieving the same brightness, as expressed by WLR.

There is broad agreement throughout the diamond trade that cutting a diamond for maximum weight yield without consideration of the final appearance constitutes unacceptably poor manufacturing. Nevertheless, the disagreement over which proportions yield the best-looking diamonds fuels the debate as to how to maximize both weight yield and appearance. Although brilliance is only one aspect of overall diamond appearance, our results indicate that for the same piece of rough, it is possible to attain greater yield with the same WLR.

Figure C-1. Starting with a rough diamond that is a highly symmetric octahedron, one can manufacture a stone with the high crown typical of the "Ideal" and obtain a good yield.

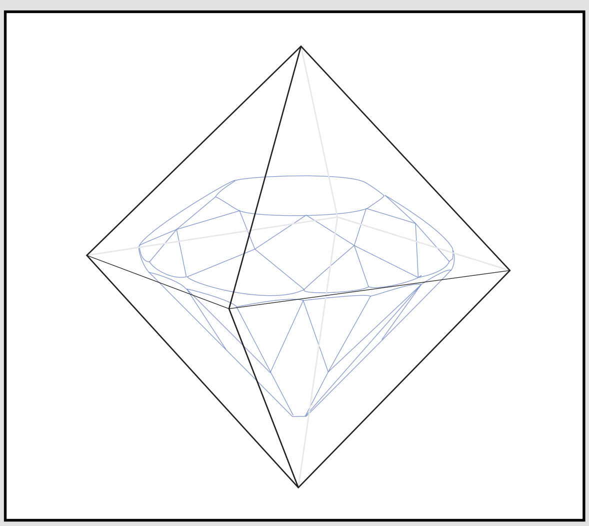
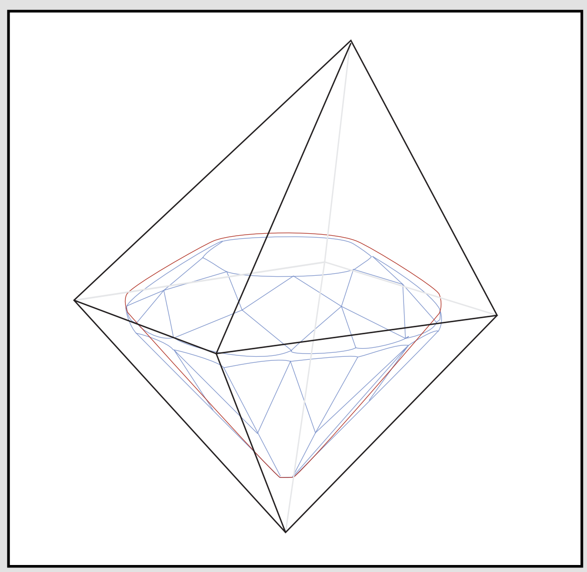


Figure C-2. Other sets of proportions, particularly slightly lower crown angles, often give the best yield from commonly encountered asymmetric octahedral diamond rough, with equivalent brightness. This yield can be significantly lower when such rough is fashioned to "Ideal" proportions.



ways that faceted shape and proportions affect the face-up appearance of fancy-colored diamonds. In addition, we hope to address the effects that different kinds of inclusions can have on the paths of light rays in a diamond (e.g., reflection from the surface of a "feather," or scattering from a cloud of pinpoints), and the additional light loss that results from poor surface finish.

CONCLUSION

In this first report of the results of our research on cut proportions, we have presented a mathematical model of the round brilliant diamond that describes this shape in terms of eight proportion parameters. It also incorporates the physical factors that affect how light interacts with a faceted diamond. At present, the "virtual" diamonds we have examined are all colorless, flawless round brilliants with mathematically perfect symmetry and polish; they vary only in their proportions. We created digital images of some of these virtual diamonds that reproduce the key features of actual diamonds (again, see figures 2, 4, 5, 7, and 12).

In this report, we have focused on brilliance, which was considered the main factor of diamond appearance in most previous analyses of the round brilliant diamond—from Tolkowsky in 1919 to Shannon and Wilson in 1998. We have quantified brilliance on the basis of weighted light return (WLR). After calculating WLR values for more than 20,000 proportion combinations, we found that the relationship between brilliance and the three primary proportion parameters (crown angle, pavilion angle, and table size) is complex, and that there are a number of proportion combinations that yield high WLR values. We also discovered that there are some commercial proportion combinations that produce rather low WLR values (again, see figure 1 and Box B). Comparisons to actual diamonds support our premise that WLR captures the essence of brilliance.

Our model differs from its predecessors in one or more of three ways: (1) it is three-dimensional; (2) it uses the most detailed existing data on the properties of a colorless diamond; and (3) it uses an averaged observer condition that takes into account the likeliest ways in which a diamond dealer or consumer looks at the stone. (The last is unique to this model.) Nevertheless, we do not consider the WLR metric we have devised to be the whole story with regard to diamond appearance.

Brilliance is only one part of the puzzle; fire and scintillation, and probably symmetry deviations and color, will also have to be analyzed before the effects of cutting on diamond appearance can be fully understood. Yet, no fashioned diamond can be considered beautiful if it lacks brilliance. We can infer from the WLR data that certain combinations of proportions will produce low light return. This is important since a round brilliant that is severely deficient in any one appearance aspect cannot be considered well cut, even if it performs well for another aspect. For example, these WLR results could be used to define proportions for which a round brilliant diamond will appear too dark; no amount of fire or pleasing scintillation would balance such darkness to produce beauty.

Ultimately, we hope to use this model to find the various ranges of proportions that clearly fail to bring out the attractive qualities of a round brilliant diamond for each appearance aspect (brilliance, fire, and scintillation). The proportion ranges that remain can be examined for balances between the different appearance aspects, and an intelligent, fact-based discussion can take place regarding which proportions produce diamonds of superior appearance. It is our opinion that any cut grading assessment devised in the absence of this broader picture is premature.

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